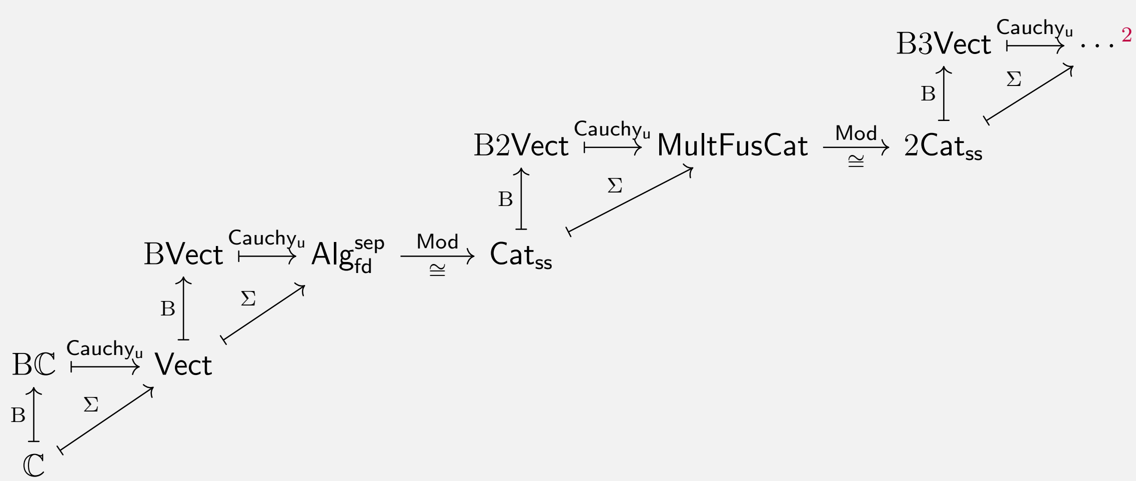
Here are 4 charts which summarize this course.

- The staircase of n-vector/Hilbert spaces [GJF19]
- The synoptic chart of tensor categories $n = 1$ and $k \leq 3$ [HPT16][1]
- The periodic table of k-tuply monoidal n-categories, $-2 \leq n \leq 2$ [BD95].
- The chart of higher categories and topological order from the 2022 AIM workshop on Higher Categories and Topological Order [Del22]

The staircase for $n\mathsf{Vect}/n\mathsf{Hilb}$ [GJF19]

Formal construction of $k\mathsf{Vect}$ from $(k-1)\mathsf{Vect}$.



Notation:

- B means take the *delooping* [BS10, §5.6], i.e., consider the monoidal k-category as a $(k+1)$-category with one object.
- $\mathsf{Cauchy_u}$ means take a unital higher Cauchy completion [GJF19].
- Σ is the composite $\mathsf{Cauchy_u} \circ \mathrm{B}$, called the *suspension*.
- Mod is the equivalence given by taking the 1- or 2-category of modules for the algebra/multifusion category respectively.

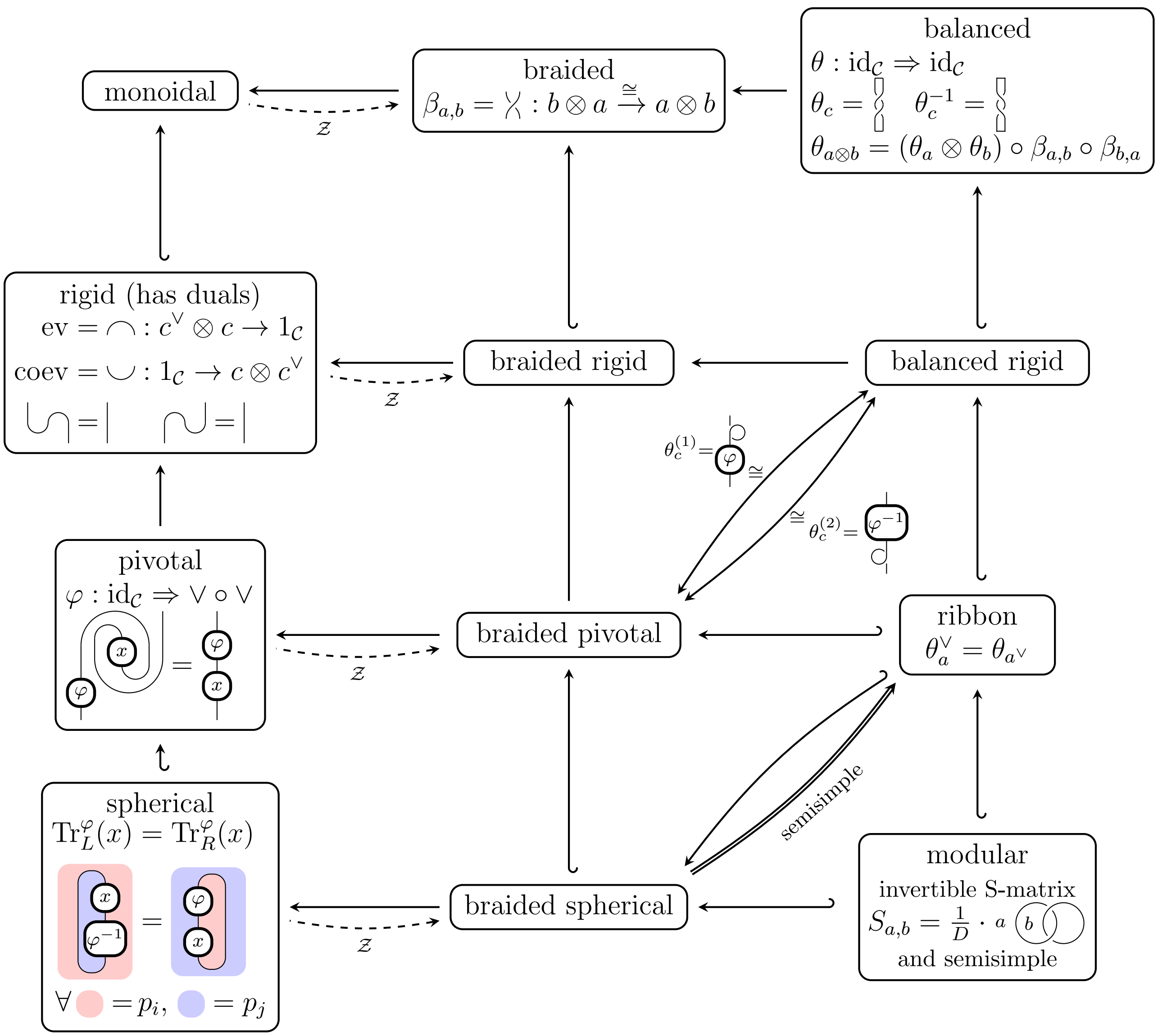
[1] Here is a great research project: Do a chart for $n = 2$ and $k \leq 4$!

[2] Here is another research project: Show that the 4-category of multifusion 2-categories goes here.

The synoptic chart of tensor categories [HPT16]

In the chart below, which is adapted from [HPT (`MR3578212`, `arXiv:1509.02937`), §2.3],

- (A) ⟶ (B) indicates that B can be obtained from A by forgetting part of the data; equivalently, A can be obtained from B by adding extra structure.
- (A) ↪ (B) indicates that A can be obtained from B by imposing extra axioms; equivalently, A is a *property* of B, and not extra structure.
- (A) $\overset{\mathcal{Z}}{\dashrightarrow}$ (B) indicates that the Drinfeld center construction goes from A to B.
- (A) ⟷ (B) indicates an equivalence between A and B.
- (A) $\overset{\text{P}}{\Longrightarrow}$ (B) indicates that A implies B assuming in addition property P.



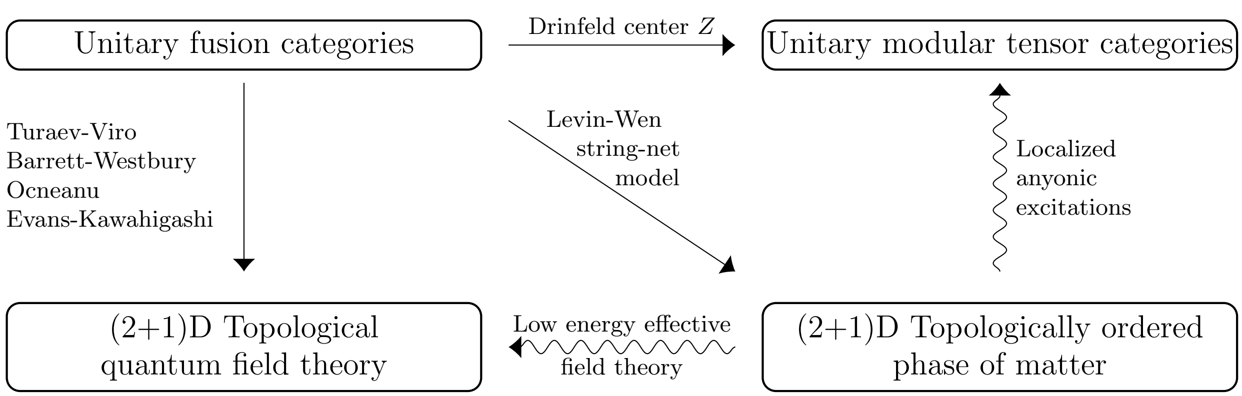
The periodic table of k-tuply monoidal n-categories [BD95, BS10]

k-tuply monoidal n-categories [BD95, BS10]. For a k-tuply monoidal n-category, being trivial at height k corresponds to extra structure on an n-category, except at height $n-1$, which is a property of an $(n+2)$-tuply monoidal n-category.

	$n=-2$	$n=-1$	$n=0$	$n=1$	$n=2$
$k=0$	$*=T$	$\{T,F\}$	set	category	2-category
$k=1$	"	$*$=T	monoid	monoidal	monoidal
$k=2$	"	"	commutative	braided	braided
$k=3$	"	"	"	symmetric	sylleptic
$k=4$	"	"	"	"	symmetric
$k=5$	"	"	"	"	"

In the chart above, we included columns for $n=-2,-1,0$, when strictly speaking, these values of n do not give categories. It is helpful to think of these levels as 'lower' categories using *negative categorical thinking* [BS10].

Chart of fusion categories and topological order [Del22]



- The topological quantum field theories constructed from unitary fusion categories are fully extended.
- The unitary modular tensor categories constructed from unitary fusion categories are achiral.

References

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