MATH 6112: ALGEBRA II
HOMEWORK #11

Due: April 22, 2019

1. Let E/F be a finite extension of finite fields. Show that the norm $N\colon E^\times \to F^\times$ is surjective.

2. (Artin-Schreier) Let F be a field of characteristic $p > 0$, and E/F a cyclic (Galois) extension of degree p. Show that E is the splitting field of $f(x) = x^p - x - a$, for some $a \in F$. [Hint: Use the additive form of Hilbert 90 to adapt the proof we gave for cyclic Kummer extensions.]

3. (Converse to #2) Let F be a field of characteristic $p > 0$, and $E = F(\alpha_1, \ldots, \alpha_t)$, with each α_i satisfying the Artin-Schreier equation $x^p - x - a_i = 0$, for some $a_i \in F$. Show that E/F is an abelian extension of exponent p.

4. Let $q = p^r$ for some $r > 0$, and $\mathbb{F}_q$ the finite field with q elements. Show that
$$G = \mathrm{Gal}(\overline{\mathbb{F}}_q/\mathbb{F}_q) \cong \widehat{\mathbb{Z}} := \varprojlim \mathbb{Z}/n\mathbb{Z},$$
and that this group is *topologically* generated by the Frobenius automorphism. (That is, G is the closure of the cyclic subgroup $\langle \mathcal{F}_q \rangle$.) Also show that G is uncountable. (So most automorphisms do not come from finite extensions!)