MATH 6112: ALGEBRA II
HOMEWORK #6

Due: February 22, 2019

1. Show that any direct product of injective modules is injective.

2. (Baer's criterion) Show that a (left) R-module Q is injective if and only if any homomomorphism from a (left) ideal $\mathfrak{a}$ to Q can be extended to a homomorphism from R to Q.

3. An R-module M is *divisible* if for every nonzero $r \in R$, the endomorphism $m \mapsto r \cdot m$ is surjective. Check that the homomorphic image of a divisible module is again divisible.

   If R is a domain, show that every injective R-module is divisible. Conversely, if R is a PID, show that every divisible module is injective. Conclude that $\mathbb{Q}$ and $\mathbb{Q}/\mathbb{Z}$ are injective abelian groups.

4. Suppose $F\colon (R - \mathbf{mod}) \to \mathcal{A}$ is a right-exact functor to an abelian category $\mathcal{A}$. (If you like, you may assume $\mathcal{A} = (S - \mathbf{mod})$ for some ring S.) Show that there are natural isomorphisms $L_0F \cong F$. Similarly, if F is left-exact, show that R^0F and F are naturally isomorphic.