HOMEWORK #2

Due: September 25, 2026

1. Let A be a noetherian ring, and $x \in A$ a non-unit and non-zerodivisor. Show that
$$\operatorname{Ass}_A(A/xA) = \operatorname{Ass}_A(A/x^nA)$$
for all $n > 1$.

2. Let $A \subseteq B$ be domains, with the same field of fractions. Show that if B is faithfully flat over A, then $A = B$.

3. Let N_λ, $\lambda \in \Lambda$, be an indexed family of submodules of a fixed A-module N. Let M be a flat A-module. Construct an example demonstrating that
$$\left(\bigcap_{\lambda\in\Lambda} N_\lambda\right) \otimes M \quad \text{and} \quad \bigcap_{\lambda\in\Lambda} (N_\lambda \otimes M)$$
need not be equal when Λ is infinite.

4. (Eisenbud, 3.10.3) If A is Noetherian and M and N are finite A-modules, show that
$$\operatorname{Ass}_A \operatorname{Hom}_A(M, N) = \operatorname{Supp} M \cap \operatorname{Ass}_A N.$$
(*Hint:* Show that it suffices to assume A is local and prove that the maximal ideal is in the set on the left-hand side iff it is in the set on the right-hand side. You will need NAK.)
Taking $M = A/I$, and setting $(0 :_N I) = \{n \in N \mid In = 0\}$, show that $\operatorname{Hom}_A(M, N) \cong (0 :_N I)$, and thus
$\operatorname{Ass}_A(0 :_N I) = \operatorname{Ass}_A N \cap \{\mathfrak{p} \subset A \mid \mathfrak{p} \text{ is a prime ideal and } I \subset \mathfrak{p}\}$.