HOMEWORK #4

Due: October 13, 2021

1. Consider the morphism $F\colon \mathbb{A}^2 \to \mathbb{A}^2$ given by $(x, y) \mapsto (x, xy)$ (as in HW #1). Show that F is birational, so its image is dense; also show that this image is *not* isomorphic to any (quasi-projective) variety. (Hint: use the theorem on dimension of fibers.)

2. Let $N = (n+1)(m+1)-1$. Prove that the Segre variety $\sigma(\mathbb{P}^n \times \mathbb{P}^m) \subseteq \mathbb{P}^N$ is not contained in any proper linear subspace of $\mathbb{P}^N$.

3. Consider the two projections $\mathbb{P}^1 \times \mathbb{P}^1 \to \mathbb{P}^1$, i.e., $\pi_1(p, q) = p$ and $\pi_2(p, q) = q$. Prove that $\pi_1(X) = \pi_2(X) = \mathbb{P}^1$ for any closed irreducible subvariety $X \subseteq \mathbb{P}^1 \times \mathbb{P}^1$, with three exceptions: (1) X is a point $(p_0, q_0) \in \mathbb{P}^1 \times \mathbb{P}^1$; (2) X is a line $\{p_0\} \times \mathbb{P}^1$; or (3) X is a line $\mathbb{P}^1 \times \{q_0\}$.

4. Consider the map $f\colon \mathbb{A}^1 \to X = \{y^2 = x^3\} \subseteq \mathbb{A}^2$ given by $t \mapsto (t^2, t^3)$. Is this map $f\colon \mathbb{A}^1 \to X$ finite?