A shot can be viewed as a Bernoulli trial having two outcomes, *success* (hitting the bullseye) with probability 0.2 and *failure* (missing the bullseye) with probability 0.8. The game is then a sequence of 25 independent such Bernoulli trials. Let \( N \) be the number of successes. Then \( N \) has a binomial distribution \( B(25,0.2) \). Let \( R \) be the reward and \( X \) be the net gain of the shooter. Then \( X = R - 25 \) (the reward minus the cost of the game).

\[
\begin{array}{ccccccccccc}
N & 0 & 1 & \cdots & 4 & 5 & 6 & \cdots & k & \cdots & 24 & 25 \\
p_k & p_0 & p_1 & \cdots & p_4 & p_5 & p_6 & \cdots & p_k & \cdots & p_{24} & p_{25} \\
R & 0 & 0 & \cdots & 0 & 10 & 15 & \cdots & 5(k-3) & \cdots & 105 & 110 \\
X & -25 & -25 & \cdots & -25 & -15 & -10 & \cdots & 5k-40 & \cdots & 80 & 85 \\
\end{array}
\]

where \( p_k = \binom{25}{k}(.2)^k(.8)^{25-k} \).

(We really do not need the last row because \( X = R - 25 \).)

Since
\[
E(X) = E(R - 25) = E(R) - E(25) = E(R) - 25
\]
all we need is to find \( E(R) \). We are going to find a relation between \( R \) and \( N \).

From the column where \( N = 5 \) on, we see \( R = 5(N - 3) = 5N - 15 \). That is \( R \) is almost equal to \( 5N - 15 \) except the first a few values corresponding to \( N = 0, 1, 2, 3, \) and \( 4 \). By adjusting the first a few values of \( 5N - 15 \) by a \( C \) as in the following chart, we get an expression of \( R \) as the sum of \( 5N - 15 \) and \( C \):

\[
\begin{array}{ccccccccccc}
N & 0 & 1 & 2 & 3 & 4 & 5 & \cdots & k & \cdots & 24 & 25 \\
5N-15 & -15 & -10 & -5 & 0 & 5 & 10 & 15 & \cdots & 5k-15 & \cdots & 105 & 110 \\
C & 15 & 10 & 5 & 0 & -5 & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 \\
R & 0 & 0 & 0 & 0 & 0 & 10 & 15 & \cdots & 5(k-3) & \cdots & 105 & 110 \\
\end{array}
\]

Now \( R = (5N - 15) + C \) implies
\[
E(R) = E(5N - 15 + C) = 5E(N) - 15 + E(C).
\]

\( N \) is a binomial distribution \( B(25, .2) \), so \( E(N) = 25 \cdot (.2) = 5.1 \).

Recalling \( C \) in the chart above, we have (note that \( p_k = \binom{25}{k}(.2)^k(.8)^{25-k} \))
\[
E(C) = 15p_0 + 10p_1 + 5p_2 + 0p_3 + (-5)p_4 + \cdots (\text{computing}) \cdots = -286441042
\]

Substituting this value of \( E(C) \) and \( E(N) = 5 \) into (2), we get \( E(R) = 9.713558958 \). Substituting this value of \( E(R) \) into (1), we get \( E(X) = -15.28644104 \).

\[1\] If \( X \) is \( B(n, p) \), then \( E(X) = np \).