Math 532 Test 1 Name: ______________________________

Spring 2008

1. Find Var(X) for the random variable X with density function $f(x) = \begin{cases} x/2, & \text{if } 0 \leq x \leq 2 \\ 0, & \text{otherwise.} \end{cases}$

Solution

$$E(X) = \int_0^2 x f(x)\, dx = \frac{1}{2}\int_0^2 x^2\, dx = \frac{4}{3}$$

$$E(X^2) = \int_0^2 x^2 f(x)\, dx = \frac{1}{2}\int_0^2 x^3\, dx = 2$$

$$Var(X) = E(X^2) - [E(X)]^2 = 2 - \left(\frac{4}{3}\right)^2 = \frac{2}{9}.$$

Answer: 2/9

2. Let X be a continuous random variable with density function

$$f(x) = \begin{cases} 6x(1-x), & \text{for } 0 < x < 1 \\ 0, & \text{otherwise.} \end{cases}$$

Find $P[|X - \frac{1}{2}| > \frac{1}{4}]$.

A. .0521 B. .1563 C. .3125 D. .5000 E. .8000

Solution $|x - \frac{1}{2}| > 1/4 \Longrightarrow x > 3/4$ or $x < 1/4$. So

$$P\left(\left|X - \frac{1}{2}\right| > \frac{1}{4}\right) = \int_0^{1/4} f(x)\, dx + \int_{3/4}^1 f(x)\, dx = .3125.$$

Answer: C

3. Let A, B, and C be mutually independent events such that $P[A] = .5$, $P[B] = .6$ and $P[C] = .1$. Calculate $P[A' \cup B' \cup C]$.

A. .69 B. .71 C. .73 D. .98 E. 1.00

Solution

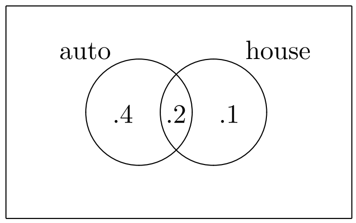
$$P(A' \cup B' \cup C) = P[(A \cap B \cap C')'] = 1 - P(A \cap B \cap C') == 1 - P(A)P(B)P(C') = .73.$$

Answer: C

4. A marketing survey indicates that 60% of the population owns an automobile, 30% owns a house, and 20% owns both an automobile and a house. Calculate the probability that a person chosen at random owns an automobile or a house, but not both.

A. .4 B. .5 C. .6 D. .7 E. .9

Solution



$$.4 + .1 = .5$$

Answer: B

5. A box contains 4 red balls and 6 white balls. A sample of size 3 is drawn without replacement from the box. What is the probability of obtaining 1 red ball and 2 white balls, given that at least 2 of the balls in the sample are white?

A. 1/2 B. 2/3 C. 3/4 D. 9/11 E. 54/55

Solution Let A represent the event of drawing one red and two white balls, and let B represent the event of drawing at least two white balls. We have

$$A \cap B = A, \quad P(A) = \frac{\binom{6}{2}\binom{4}{1}}{\binom{10}{3}} = \frac{1}{2}, \quad P(B) = P(1 \text{ red, } 2 \text{ whites}) + P(3 \text{ whites}) = \cdots = \frac{2}{3}$$

and so

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)}{P(B)} = \frac{3}{4}.$$

Answer: C

6. Let X_1, X_2, and X_3 be three *independent* continuous random variables each with density function

$$f(x) = \begin{cases} \sqrt{2} - x, & \text{for } 0 < x < \sqrt{2} \\ 0, & \text{otherwise.} \end{cases}$$

What is the probability that exactly 2 of the 3 random variables exceeds 1?

A. $3/2 - \sqrt{2}$ B. $3 - 2\sqrt{2}$ C. $3(\sqrt{2}-1)(2-\sqrt{2})^2$
D. $(3/2 - \sqrt{2})^2(\sqrt{2} - 1/2)$ E. $3(3/2 - \sqrt{2})^2(\sqrt{2} - 1/2)$

Solution (We went over this in class)

$$P(\text{exactly } 2 > 1) = 3P(X_1 > 1, X_2 > 1, X_3 < 1) = 3P(X_1 > 1)^2 P(X_3 < 1) =$$

$$= 3\left(\int_1^{\sqrt{2}}\right)^2 \left(\int_0^1\right) = 3(3/2 - \sqrt{2})^2(\sqrt{2} - 1/2).$$

Answer: E

7. The board of directors of a corporation wishes to purchase "headhunter insurance" to cover the cost of replacing up to 3 of the corporation's high-ranking executives, should they leave during the next year to take another job. The board wants the insurance policy to pay $1,000,000\times K^2$, where $K = 0, 1, 2$ or 3 is the number of the three executives that leave whithin the next year. An actuary analyzes the past experience of the corporation's retention of executives at that level, and estimates the following probabilities for the number who will leave: $P[K = 0] = .8$, $P[K = 1] = .1$, $P[K = 2] = P[K = 3] = .05$. Find the expected payment the insurer will make for the year on this policy.

A. 250,000 B. 500,000 C. 750,000 D. 1,000,000 E. 2,000,000

Solution We have

$$E(K^2) = 0^2 \cdot P(K = 0) + 1 \cdot P(K = 1) + 2^2 \cdot P(K = 2) + 3^2 \cdot P(K = 3) =$$

$$= 1 \cdot (.1) + 4 \cdot (.05) + 9 \cdot (.05) = .75.$$

So, Expected payment $= E(1000000K^2) = 1000000E(K^2) = 750000.$

Answer: C

8. Two bowls each contains 5 black and 5 white balls. A ball is chosen at random from bowl 1 and put into bowl 2. A ball is then chosen at random from bowl 2 and put into bowl 1. Find the probability that bowl 1 still has 5 black and 5 white balls.

A. 2/3
B. 3/5
C. 6/11
D. 1/2
E. 6/13

Answer: C

9. A test for a disease correctly diagnoses a diseased person as having the disease with probability .85. The test incorrectly diagnoses someone without the disease as having the disease with a probability of .10. If 1% of the people in a population have the disease, what is the chance that a person from this population who tests positive for the disease actually has the disease?

A. .0085 B. .0791 C. .1075 D. .1500 E. .9000

Solution Let H represent a randomly chosen person has the disease, and let T represent a person is tested as having the disease. The given conditions can be written as

$$P(T|H) = .85, \quad P(T|H') = .1, \quad P(H) = .01.$$

We have $P(T) = P(T|H)P(H) + P(T|H')P(H') = (.85)(.01) + (.1)(.99) = .1075$, and

$$P(H|T) = \frac{P(T|H)P(H)}{P(T)} = \frac{(.85)(.01)}{.1075} = .0790697674... \approx .0791.$$

Answer: B