Math 630, Problem Set 1

1. Given: \( l_1 = 9700, q_1 = q_2 = .02, q_4 = .026, \) and \( d_3 = 232. \) Determine the number of survivors to age 5.

2. Given \( t|_q x = .10 \) for \( t = 0, 1, \ldots, 9, \) calculate \( 2p_{x+5}. \)

3. Given \( S_0(x) = \frac{10000-x^2}{10000} \) for \( 0 \leq x \leq 100, \) calculate \( q_{32}. \)

4. A person age 70 is subject to the force of mortality \( \mu_{70+t} = \begin{cases} .01, & t \leq 5 \\ .02, & t > 5 \end{cases}. \) Calculate \( 20p_{70}. \)

5. Given \( \mu_x = \frac{2x}{10000-x^2} \) for \( 0 \leq x \leq 100, \) determine \( q_x. \)

6. For a standard insured, the force of mortality \( \mu_{30+t} \) for \( 0 \leq t \leq 1 \) and \( p_{30} = .95. \) For a preferred insured, the force of mortality is \( \mu_{30+t} - c \) for \( 0 \leq t \leq 1. \) Determine \( c \) such that \( q_x \) is reduced by 25%.

7. A life table for severely disabled is created by modifying an existing life table by doubling the force of mortality at all ages. In the original table, \( q_{75} = .12. \) Calculate \( q_{75} \) in the modified table.

8. In DML with \( \omega = 105, \) calculate \( 10|_{20}q_{25}. \)

9. In a DML model, \( q_{10} = 1/45. \) Determine \( \mu_{10}. \)

10. A life is subject to constant force of mortality \( \mu. \) Given that \( e_{50} = 24, \) determine \( \mu. \)

11. Given that mortality follows DML and \( \hat{e}_{30} = 30, \) find \( q_{30}. \)

12. Mortality follows DML. \( \text{Var}(T_{15}) = 675. \) Calculate \( \hat{e}_{25}. \)

13. Given that \( S_0(x) = e^{-0.05x}, \) find \( \hat{e}_{30}. \)

14. Hens lay an average of 30 eggs each month until death. The survival function for hens is \( S_0(m) = 1 - \frac{m}{72} \) where \( m \) is in months. 100 hens have survived to age 12 months. Calculate the expected total number of eggs to be laid by these 100 hens in their remaining lifetimes.

15. Given: \( e_{35} = 49 \) and \( p_{35} = .995. \) If \( \mu_x \) is doubled for \( 35 \leq x \leq 36, \) what is the revised value of \( e_{35}? \)

16. Given that \( \frac{1}{4}q_{x+\frac{3}{4}} = \frac{3}{31} \) and mortality is UDD from age \( x \) to \( x + 1, \) calculate \( q_x. \)

17. Deaths are UDD between integral ages, \( q_x = 0.10, \) and \( q_{x+1} = 0.15. \) Calculate \( 0.3|_{0.5}q_{x+0.4}. \)
18. A mortality study is conducted for the age interval \([x, x+1]\). If a constant force of mortality applies over the interval, \(\frac{1}{4} q_{x+1}^* = 0.05\). Calculate \(0.25 q_{x+0.1}\) assuming UDD over the interval.

19. Given \(\begin{array}{c|ccccc} x & 90 & 91 & 92 & 93 & 94 \\ \hline q_x & 0.10 & 0.12 & 0.13 & 0.15 & 0.16 \end{array}\) and 
\[
q_x = 0.5q_x, \quad q_{x+1} = 0.75q_{x+1}, \quad l_{[91]} = 10000,
\]
find \(l_{[90]}\) if the select period is 2 years.

20. Given a 2-year select period and the table
\[
\begin{array}{c|ccc}
 x & 1000q_x & 1000q_x+1 & 1000q_x+2 \\
\hline
40 & .438 & .574 & .699 \\
41 & .453 & .599 & .738 \\
42 & .477 & .634 & .790 \\
43 & .510 & .680 & .856 \\
44 & .551 & .737 & .937 \\
\end{array}
\]

calculate \(1000_{1/2}q_{[41]}\).

21. Given a 2-year select period
\[
\begin{array}{c|ccc}
 x & l_x & l_{x+1} & l_{x+2} \\
\hline
24 & -- & -- & 42,683 \\
25 & -- & -- & 35,000 \\
26 & -- & -- & 26,000 \\
\end{array}
\]
and \(q_{x+1} = 1.5q_{x+1}, \quad q_{x+2} = 1.2q_{x+1+1}\). calculate \(l_{[26]}\).

22. Mortality follows Gompertz Law with \(B = 0.002\) and \(c = 1.03\), find \(t\) such that \(t p_{45} = 0.6\).

Answers
1. 8848; 2. 3/5; 3. 0.007242; 4. 0.70469; 5. \(\frac{2x+1}{1000-x^2}\); 6. 0.013072; 7. 0.2256; 8. 1/4; 9. 1/45; 10. .040822; 11. 1/60; 12. 40; 13. 20; 14. 90000; 15. 48.755; 16. 0.3; 17. 0.059375; 18. 0.04725; 19. 11279.53; 20. 1.336; 21. 36944; 22. 37.12674.
Solutions to Selected Problems

4.
\[ 20p_{70} = \exp \left( - \int_0^{20} \mu_{70+t} \, dt \right) = \exp \left( - \int_0^5 \mu_{70+t} \, dt - \int_5^{20} \mu_{70+t} \, dt \right) = \exp \left( - \int_0^5 0.01 \, dt - \int_5^{20} 0.02 \, dt \right) = e^{-0.35}. \]

11. \( \hat{e}_{30} = 30 \Rightarrow \frac{\omega - 30}{2} = 30 \Rightarrow \omega = 90. \) So \( q_{30} = \frac{1}{\omega - 30} = \frac{1}{60}. \)

13. \( S_0(x) = e^{-0.05x} \Rightarrow \text{CFM with } \mu = 0.05. \) So \( \hat{e}_{30} = \frac{1}{\mu} = \frac{1}{0.05} = 20. \)

18. First note that \( 0.35p_x = 0.1p_x \cdot 0.25p_{x+0.1} \quad (1) \)
Under CFM, \( 0.25q_{x+0.1} = 0.05 \Rightarrow 0.25p_{x+0.1} = 0.95 \Rightarrow e^{0.25\mu} = 0.95 \Rightarrow e^\mu = 0.95^4 = p_x \)
Under UDD, \( (1) \) becomes \( 1 - (0.35)q_x = (1 - 0.1q_x)(0.25p_{x+0.1}). \)
Using \( q_x = 1 - p_x = 1 - 0.95^4 \) in the last equation, we get \( 0.25p_{x+0.1} = 0.04725. \)

20. We have \( 1000 \cdot 1[2q_{[41]}] = 1000 \cdot p_{[41]} \cdot 2q_{[41]+1}. \) Further more, \( p_{[41]} = 1 - q_{[41]} \) and \( 2q_{[41]+1} = 1 - 2p_{[41]+1} = 1 - p_{[41]+1} \cdot p_{41+2} = 1 - (1 - q_{[41]+1})(1 - q_{41+2}) = q_{[41]+1} + q_{41+2} - q_{[41]+1} \cdot q_{41+2}. \)
Then use the values given in the table.

21. (Sketch)

- First note that \( q_{[x]+2} = q_{x+2}, \) so we are actually given that \( q_{x+2} = 1.2q_{[x+1]+1}. \)
- Using the given \( l_{26}, l_{27} \) and \( l_{28} \), we can find \( p_{26} \) and \( p_{27}. \)
- Using \( p_{26} \) and \( p_{27} \) with \( q_{x+2} = 1.2q_{[x+1]+1}, \) we can find \( p_{[25]+1} \) and \( p_{[26]+1}. \)
- Using \( p_{[25]+1} \) and \( p_{[26]+1} \) with \( q_{[x]+1} = 1.5q_{[x+1]}, \) we can find \( p_{[26]} \) and \( p_{[27]} \).
- Using the given \( l_{28} = l_{26} + 2, p_{[26]+1}, \) and \( p_{[26]}, \) we can find \( l_{[26]}. \)

22.
Under Gompertz’s law, \( \mu_x = Bc^x = (0.002)(1.03)^x. \) So,
\[ tP_{45} = \exp \left( - \int_0^t (0.002)(1.03)^{45+s} \, ds \right) = \exp \left( - \frac{0.003}{\ln 1.03} \cdot 1.03^{45} \cdot (1.03^t - 1) \right) = 0.6 \]
Solving for \( t \) in the last equation.