Guaranteed Benefits

Financial Math Seminar
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Hedging Guarantees

Hedging insurance products combines elements of both *actuarial science* and *quantitative finance.*

Why? The financial risk and the insurance risk embedded in guaranteed benefits are inseparable

- These risk management challenges don't have well-documented, easy-to-implement answers

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Annuities

What is an annuity?

- The policyholder makes either one lump sum payment or a series of payments
- The insurance company pays out periodic payments in return, beginning immediately or in the future
- The payments to the policyholder may be for a definite period (i.e., 20 years) or for an indefinite period (until death)
- Typically provide tax advantages

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Types of Annuities

Fixed Annuity

The policyholder's account value grows at a fixed rate of return, which is guaranteed by the insurance company

Variable Annuity

The policyholder chooses from among investment options, and the account value grows according to their performance

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Products Under Management

Fixed Annuity

- EIA: Equity Indexed Annuity

Variable Annuity

- GMAB: Guaranteed Minimum Accumulation Benefit
- GMWB: Guaranteed Minimum Withdrawal Benefit
- GMDB: Guaranteed Minimum Death Benefit

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Equity Indexed Annuity

The change in a policyholder's account value is linked to the change in a specified index over the same period of time.

It may sound simple enough, but there are many variations in product design.

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Important EIA Features

Indexing methods include:

- Annual Reset
- High Water Mark
- Point-to-Point

How much of the index performance is applied?

- Participation Rate
- Cap

What about principal protection?

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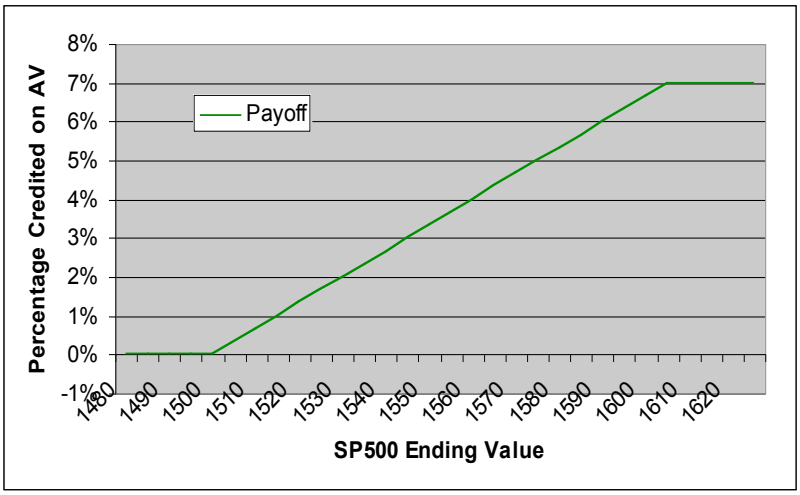
EIA Payoff



Payoff diagram:

- Indexed to SP500
- Annual reset
- Capped at 7%
- Struck at 1500

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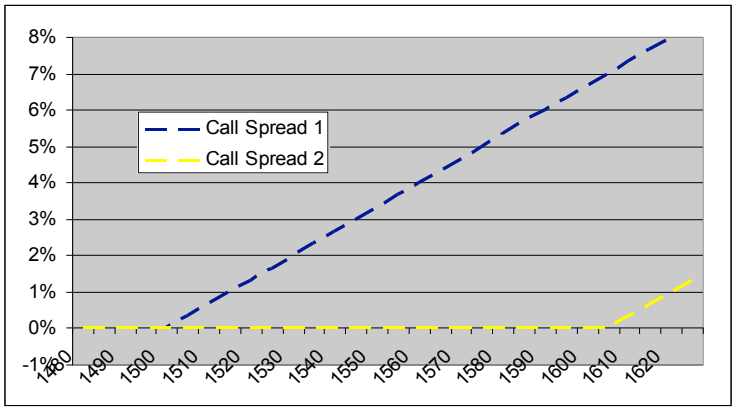
Static Hedging

Static Hedging is a "buy and hold" strategy:

Purchase an asset and hold it until maturity, in hopes that it will replicate the cash flows of your liability.



Consider a portfolio of two calls:

Buy one call struck at 1500

Sell one call struck at 1605 (1.07*1500)

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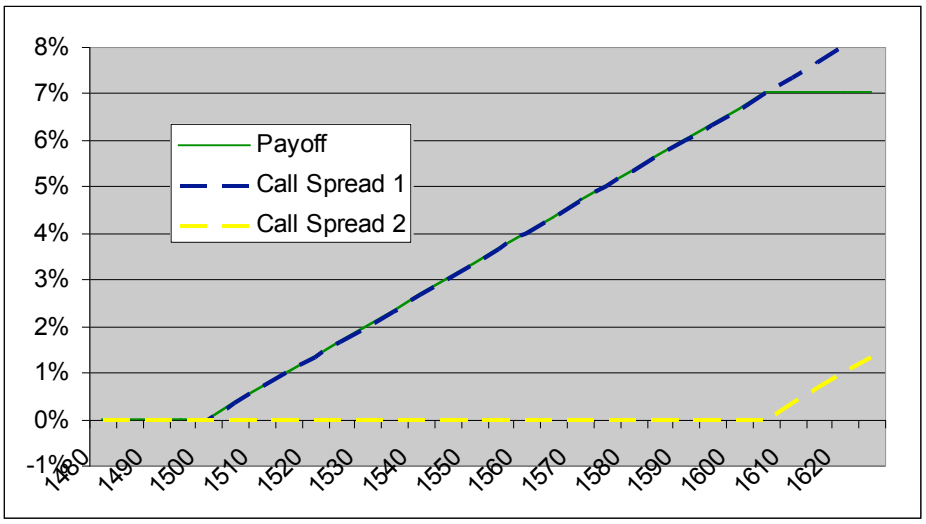
Payoff of portfolio = Payout to Policyholder



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Complications

Jellybean Problem

Just as you cannot walk into a candy store and buy one jellybean, an insurance company cannot buy a separate call spread for each policy from an investment bank. (Candy stores and investment banks like to do “bulk” business!)

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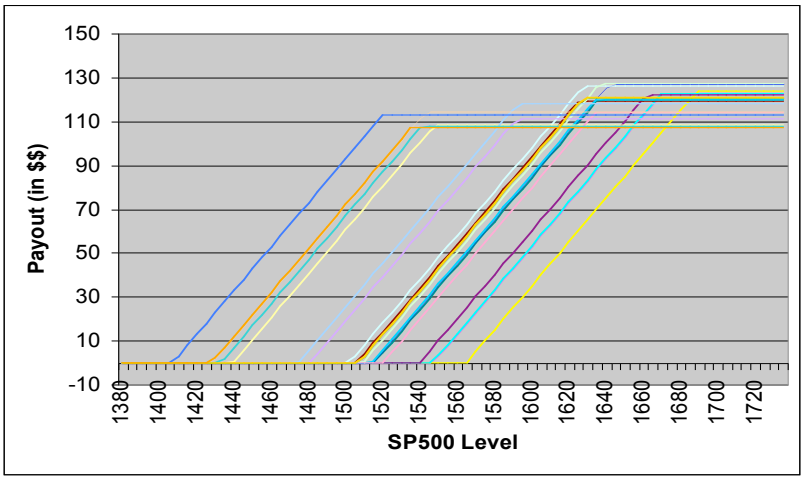
Complications



To achieve the necessary scale, several large call spreads are purchased at the end of the quarter to cover an entire quarter’s worth of new business.

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Complications

How would one find an **optimal portfolio** of call spreads, given that each policy is struck at different points on the SP500, and with different caps?

Also, how is the liability managed until the end of the quarter comes around?

(Hint: dynamic hedging)

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The Spectrum of Hedge Strategies

Static: buy and hold assets

Example: to hedge CPP, go out and buy a matching put option

Dynamic: buy and sell assets to match sensitivities of the liabilities

Example: to offset exposure to movements in equity market, buy and sell SP500 futures

Most hedge programs require a skillful combination!

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GMAB: Guaranteed Minimum Accumulation Benefit

Insurance company promises to **return the policyholder's initial premium** at the end of the AB period (benefit maturity)

- **When do we pay out?** If the account value is below the initial premium at the end of the AB period, the insurance company is on the hook for the difference

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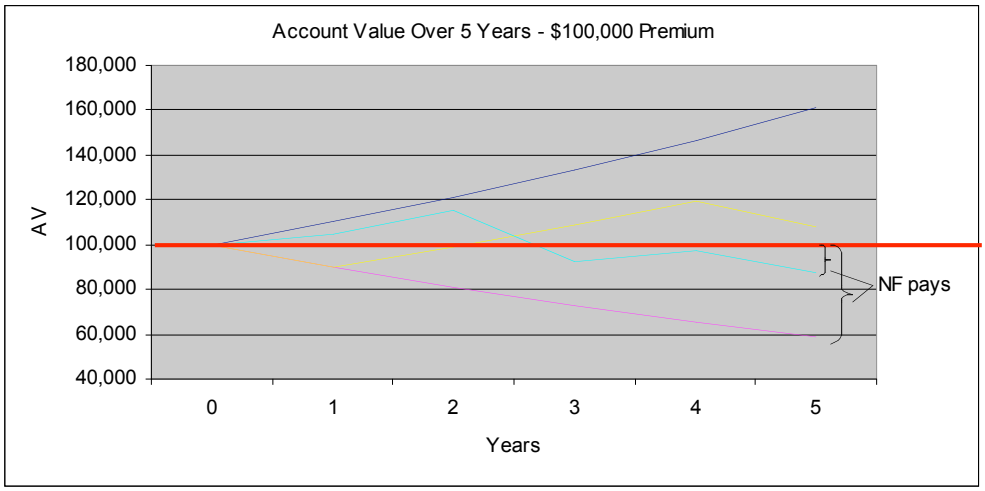
GMAB Payoff: Several Examples



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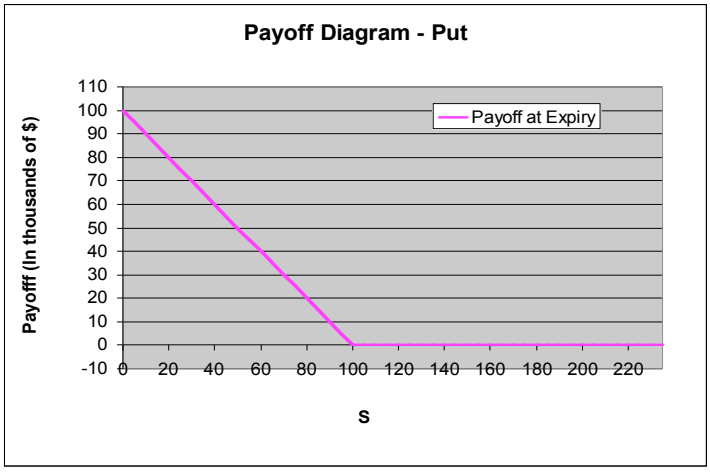
Payoff Diagram



A put is a financial option that gives you the right, but not the obligation, to sell a defined asset at a specified price at a future date.

When the put expires, if the price of the asset is below the "strike price," the put holder will exercise the option, and profit by the difference.

This is exactly the same as the policyholder's payoff for GMAB.

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The GMAB – Put Relationship

Essentially, GMAB is like a common financial option: the put

- **Valuation and calculation of sensitivities are simple**; it can even be done formulaically with Black-Scholes
- **Hedge strategy is straightforward;** you can buy back similar put options from investment banks

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Complications

Base Lapses

A policyholder can lapse at any time, although not without penalty.

How do we model lapses? Should all policies receive the same lapse assumption? What characteristics might make policyholders more or less likely to lapse?

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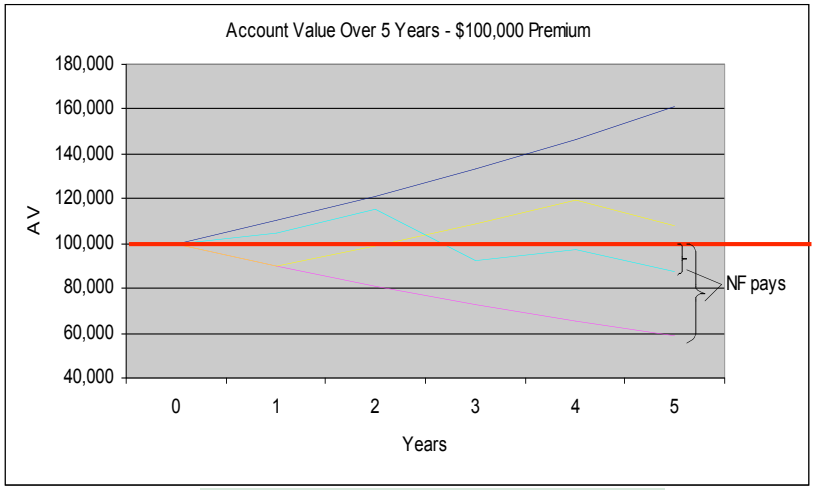
Dynamic Lapsation

Dynamic lapses account for the idea that the "In-the-Moneyness" of the benefit has an impact on the policyholder's decision to lapse.



In which scenario is lapsation more likely:

The blue scenario, or the magenta scenario?

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Next, the GMWB-for-life

Insurance company guarantees that the policyholder will ***never run out of money***, as long as they follow the specified withdrawal schedule

- When do we pay out?
  - The initial payments come out of the policyholder's account value. Once the account value hits zero, we are on the hook for these payments – for the rest of their life!
- Implications for hedging?
  - **Valuation and calculation of sensitivities is complicated.** It cannot be done formulaically, so we must use simulation.

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How are withdrawals determined?

Annual Withdrawal = Benefit Base x Withdrawal%

Benefit Base is initially defined as the starting account value. It grows over the life of the contract based on specific rules of the GMWB.

Withdrawal % is determined by the attained age of the policyholder at the time of the first withdrawal.

Once the first withdrawal is taken, both the Benefit Base and the Withdrawal % are **locked down** for the remainder of the contract.

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Benefit Base

Say the Benefit Base grows with both a ratchet and a 10 year, 5% simple rollup guarantee.

- On the first contract anniversary
  BB_1 = *Max*(Current AV, 1.05*Initial AV).

- On each subsequent anniversary thru the tenth anniversary
  BB_t = *Max*(Current AV, Initial AV +0.05*t*Initial AV).

- After the tenth anniversary, the Benefit Base is set to
  BB_t = *Max* (Current AV, BB_{t-1}).

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Consider an example

Let a policyholder initially invest $100k in a GMWB. The Benefit Base is set to $100k.
(BB_0 = $100k)

- At the end of the first year, the market is down, so the Current AV at the end of year one is $98k.
  BB_1 = Max($98k, $105k) = $105k
- The market rallies in year two, and the Current AV at the end of year two is $112.
  BB_2 = Max($112k, $110k) = $112k

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Withdrawal Percentage

- The lifetime withdrawal percentage is based on attained age at the time of the first withdrawal.
- The percentage is then locked in for the remainder of the contract.

Attained Age	Lifetime Withdrawal %
45 thru 59.5	4%
59.5 thru 66	5%
67 thru 71	5.5%
72 thru 80	6%
81 and older	7%

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Option Types: Exercise Feature

European

Can only be exercised **on the expiry date**

American

Can be exercised on **any date prior to expiry**

Bermudan

Can be exercised on **specified dates prior to expiry**

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American Optionality

The policyholder may elect to begin withdrawals at any time. Some policyholders will begin immediately, while some will wait for years.

As the policyholder waits…

- The Benefit Base and Withdrawal Percentage **increase**
- The number of years the policyholder can collect withdrawals **decreases**

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Interesting Questions

Policyholder wants to know:

When is the best time to initiate withdrawals?

Insurance company needs to know:

How many policyholders will elect the optimal date to initiate withdrawals?

How do we model this?

How sensitive is our valuation to wait time assumptions?

Recall: these are NEW benefits, so there is no historical data to use for modeling

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Further complications

Spousal Continuation Benefit

- For an additional charge, spouse may continue to receive withdrawal payments after policyholder's death
- How does this effect the optimal withdrawal start date for policyholder?
- Should insurance company take election of spousal benefit into consideration when making assumptions about withdrawal start date?

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In conclusion…

EIA: Equity Indexed Annuity

Complication: Optimal Call Spread Portfolio

GMAB: G'teed Minimum Accumulation Benefit

Complication: Lapse Behavior (Base and Dynamic)

GMWB: G'teed Minimum Withdrawal Benefit

Complication: American Optionality (Policyholder chooses withdrawal start date)

Hedging Basics: Dynamic vs. Static Hedging, American vs. European Options

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Questions?

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