Mathematics 1161: Midterm Exam 1 Review

1. **Midterm Exam 1 is on 21 September at 6:00-6:55pm in McPherson Lab 1000.**
   It will cover Sections 2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 2.7, 3.1, 3.2, 3.3, 3.4, 3.5, 3.6, and 3.7.
   (Familiarity with Sections 1.3 and 1.4 is assumed.)

2. Take your **BuckID** to the exam. The use of notes, calculators, or other electronic devices is forbidden.

3. Be able to do problems from the quizzes, online homework, and written homework.

4. Save time by memorizing the derivatives of x^n, e^{kx}, and all of the trigonometric functions.

5. Several practice exercises appear on the pages that follow. Solutions will be available on Carmen in the Modules section, and we will do a few of these problems during our review session. These exercises primarily reinforce computational skills, but keep in mind that the exam will also test conceptual understanding.

6. **What do you need to know?**

   2.1 Know what a secant line is and what a tangent line is for the graph of a function. How are these related to material from Section 3.1 (average rate of change and instantaneous rate of change)?

   2.2 Know what limits, right-sided limits, and left-sided limits are. When does the two-sided limit exist, and how is it related to the one-sided limits?

   2.3 Know how to evaluate limits by doing algebra and then using direct substitution. (Useful algebra techniques include factoring, multiplying by a conjugate, and using a common denominator to add or subtract fractions.)

   2.4 Know how to recognize that a (one-sided or two-sided) limit is ∞ or $-\infty$. (Factoring and paying attention to odd vs. even powers is often useful.) How does this relate to vertical asymptotes of a graph?

   2.5 Know how to find limits at ∞ and $-\infty$. (Adjusting by the effective growth rate of the denominator can be useful, and be careful with even-numbered radicals.) How do these relate to horizontal asymptotes of a graph?

   2.6 Know the definition of continuity at a point, and know how to use it to check if a function is continuous at a point. Know how to decide which piece of a piecewise-defined function to use when evaluating a (one-sided) limit.

   2.7 Know the ϵ-δ definition of a limit, and be able to use it to write a formal prove for a limit of a low degree polynomial. Know the N-δ definition of an infinite limit. What should the definitions of the one-sided limits or one-sided infinite limits be?

   3.1 Know how to find the slope of a secant line and how to find the slope of a tangent line. Know the definition of derivative and how to use it to compute derivatives.

   3.2 What does the shape of the graph of f tell you about f'? Know that the zeros of f' correspond to horizontal tangent lines for f.

   3.3 Know how to use the rules of differentiation.

   3.4 Know how to use the Product Rule, Quotient Rule, and Extended Power Rule.

   3.5 Know the trigonometric limits in Theorem 3.11. Know the derivatives of the trigonometric functions.

   3.6 Know how to find average velocity and instantaneous velocity.

   3.7 Know how to recognize a composition of functions, and know how to use the Chain Rule to compute a derivative.

1. Find the one-sided limit $\lim_{x\to -2^+} \dfrac{x^2+5x+6}{x+2}$ as follows:

(a) Does direct substitution apply?

(b) Factoring the numerator and cancelling gives

$$\lim_{x\to -2^+} \frac{x^2+5x+6}{x+2} = \lim_{x\to -2^+} \boxed{}.$$

(c) Does direct substitution apply to the answer to part (b)?

(d) Determine the limit.

$$\lim_{x\to -2^+} \frac{x^2+5x+6}{x+2} = \boxed{}.$$

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2. Find the one-sided limits.

(a) $\lim_{x\to -2^+} \dfrac{x+2}{x^2+4x+4}$

(b) $\lim_{x\to -2^-} \dfrac{x+2}{x^2+4x+4}$

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3. Treat m as a constant. Consider the function

$$f(x) = \begin{cases} 4x+m & \text{if } 9x < 2 \\ -6x^2+2m & \text{if } x \geq 2. \end{cases}$$

(a) Evaluate the limit in terms of m.

$\lim_{x\to 2^+} f(x)$

(b) Evaluate the limit in terms of m.

$$\lim_{x \to 2^-} f(x)$$

(c) Find all values of m that make f continuous at $x = 2$ by determining the values of m such that

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x).$$

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4. Evaluate the limit.

$$\lim_{x \to 3} \frac{x^2 - 5x + 6}{x^2 - 7x + 12}$$

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5. Evaluate the limit.

$$\lim_{x \to 36} \frac{x^2 - 1296}{\sqrt{x} - 6}$$

6. Evaluate the limit.

$$\lim_{x \to 4} \left(\frac{1}{x-4} - \frac{8}{x^2-16} \right)$$

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7. Evaluate the limit.

$$\lim_{x \to 0} \frac{x-3}{x^2(x-2)}$$

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8. Evaluate the limit.

$$\lim_{x \to \infty} \frac{3-8x^4}{5+11x^4}$$

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9. Evaluate the limit.

$$\lim_{x \to -\infty} \frac{2x^4-7x^2}{5x^6+8}$$

10. Evaluate the limits.

(a) $\lim_{x\to\infty} \dfrac{\sqrt{x^6 - 6x^3 - 3}}{10x^3 + 10}$

(b) $\lim_{x\to-\infty} \dfrac{\sqrt{x^6 - 6x^3 - 3}}{10x^3 + 10}$

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11. Evaluate the following limits.

(a) $\lim_{x\to\infty} \left(\sqrt{x^2 - 2x + 1} - x\right)$

(b) $\lim_{x \to -\infty} \left(\sqrt{x^2 - 2x + 1} - x \right)$

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12. Show that each of the following functions has a horizontal asymptote by calculating the given limit.

(a) $\lim_{x \to \infty} \dfrac{-8x}{13 + 2x}$

(b) $\lim_{x \to -\infty} \dfrac{11x - 5}{x^3 + 8x - 5}$

(c) $\lim_{x \to \infty} \dfrac{x^2 - 7x - 13}{6 - 7x^2}$

(d) $\lim_{x \to \infty} \dfrac{\sqrt{x^2 + 8x}}{11 - 15x}$

(e) $\lim_{x \to -\infty} \dfrac{\sqrt{x^2 + 8x}}{11 - 15x}$

13. Find the values of c and d that make f continuous for all x.

$$f(x) = \begin{cases} 3x & \text{if } x < 1 \\ cx^2 + d & \text{if } 1 \leq x < 2 \\ 3x & \text{if } x \geq 2. \end{cases}$$

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14. Use the **definition of derivative** to compute $f'(2)$, where $f(x) = \sqrt{3 + 3x}$.

15. Use the **definition of derivative** to compute $f'(x)$, where $f(x) = 2x + \frac{2}{x}$.

Use this to find the equation of the tangent line to the graph of $y = 2x + \frac{2}{x}$ at the point $(-4, -8.5)$.

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16. Differentiate $f(x) = 5x^2 - 2x - 18$.

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17. If $f(x) = \dfrac{3x}{1 + x^2}$, find $f'(5)$.

Use this to find the equation of the tangent line to the curve $y = \frac{3x}{1+x^2}$ at the point $(5, \frac{15}{26})$.

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18. For which values of x does the graph of

$$f(x) = 4x^3 - 18x^2 - 36$$

have a horizontal tangent?

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19. Differentiate $f(x) = x^3 + 5e^{3x}$.

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20. Differentiate $f(x) = \dfrac{9x}{3x - 7}$.

21. Differentiate $g(x) = (3x^2 - 4x - 5)e^x$.

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22. Differentiate $f(x) = \dfrac{xe^x + 5}{x^2 + 12}$.

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23. Differentiate $f(x) = 5\sin x + 2\cos x$.

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24. Differentiate $f(x) = x^4 \cos x$.

25. Differentiate $f(x) = 2\sec x + 4e^x \tan x$.

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26. Evaluate the limit.

$$\lim_{x \to 0} \frac{\sin 8x}{\sin 3x}$$

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27. Evaluate the limit.

$$\lim_{\theta \to 0} \frac{\sin(7\theta)}{\tan(6\theta)}$$

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28. If a ball is thrown straight up into the air with an initial velocity of 50 ft/s, its height in feet after t seconds is given by $h(t) = 50t - 16t^2$. Find the average velocity for the time period beginning when $t = 2$ and lasting

    (a) 0.5 seconds

(b) 0.1 seconds

Calculate the instantaneous velocity of the ball when $t = 2$.

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29. The displacement (in meters) of a particle moving in a straight line is given by

$$s = t^2 - 7t + 16,$$

where t is measured in seconds.

(a) Find the average velocity over the time interval $[3, 4]$.

(b) Find the average velocity over the time interval $[4, 5]$.

(c) Find the instantaneous velocity when $t = 4$.

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30. Let $f(x) = e^{\sqrt{3x+8}}$. Find $f'(x)$.

31. Let $f(x) = -9e^{x\cos x}$. Find $f'(x)$.

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32. Let $f(x) = \sqrt{\sin(e^{x^2\sin(x)})}$. Find $f'(x)$.

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33. Use the ϵ-δ definition of a limit to prove that $\lim_{x\to -3} f(x) = 1$, where $f(x) = 2x^2 + 12x + 19$.

Given $\epsilon > 0$, list three different choices of δ that ensure $|f(x) - 1| < \epsilon$ whenever $0 < |x - (-3)| < \delta$.