Mathematics 1161: Midterm Exam 2 Study Guide

1. **Midterm Exam 2 is on 19 October at 6:00-6:55pm in McPherson Lab 1000.**
   It will cover Sections 3.8, 3.9, 3.10, 3.11, 4.1, 4.2, 4.3, 4.4, 4.5, 4.6, 4.7, and 4.9.

2. Take your **BuckID** to the exam. The use of notes, calculators, or other electronic devices is forbidden.

3. Be able to do problems from the quizzes, online homework, and written homework.

4. Save time by memorizing the derivatives of common functions (exponential, logarithmic, trigonometric, and inverse trigonometric).

5. Several practice exercises appear on the pages that follow. Solutions will be available on Carmen in the Modules section, and we will do a few of these problems during our review session. These exercises primarily reinforce computational skills, but keep in mind that the exam will also test conceptual understanding.

6. **What do you need to know?**

   3.8 Given an equation for a curve, know how to use implicit differentiation to find $\frac{dy}{dx}$. How do you find the equation of the tangent line at a point on the curve?

   3.9 Know the derivatives of exponential and logarithmic functions (with base a). Know how to do logarithmic differentiation; when is it useful?

   3.10 Know the derivatives of the inverse trigonometric functions.

   3.11 Be ready to solve a related-rates problem. Remember: (1) make an in-motion sketch, (2) assign variables so that you can write the "given", "want", and "at" information neatly, (3) find an equation that relates the "given" and "want" variables, (4) take the derivative of both sides with respect to t (using the Chain Rule as necessary!), (5) use the "given" and "at" information to solve for the "want" rate, and (6) check your units and make sure that your answer has the appropriate sign for how the question is worded.

   4.1 Know how to find critical points. How do you find the absolute extrema for a continuous function on a closed interval? (Compare the values of the function at the critical points in the interval and at the endpoints of the interval.)

   4.2 How do you use the derivative to find the intervals on which a function is increasing or decreasing? Be able to use the First Derivative Test to identify local extrema. How do you use the second derivative to find the intervals on which as function is concave up or concave down? What is an inflection point? How do you find inflection points?

   4.3 Understand what the items from Section 4.2 mean for the shape of the graph of a function.

   4.4 Be ready to solve an optimization problem. Remember: (1) make a sketch, (2) assign variables so that you can write the objective function and the constraint(s) neatly, (3) use the constraint(s) to rewrite the objective function as a function of a single variable, (4) determine the interval of interest for the function, and (5) use calculus to find the absolute maximum or minimum.

   4.5 Know the formula for the linear approximation to f at a, and know how to use it to estimate $f(x)$ for values of x near a. Differentials will not be on this exam (but we will revisit them later in the semester).

   4.6 Know when Rolle's Theorem and the Mean Value Theorem can be applied, and know what each tells you.

   4.7 Be able to recognize indeterminant forms and how to handle each type using L'Hôpital's Rule.

   4.9 Be able to recognize the derivatives of common functions in order to find antiderivatives. Know how to use the Power, Constant Multiple, and Sum Rules for Indefinite Integrals. Be able to solve some initial value problems.

1. Let $f(x) = x^2 \ln(9 - 5x^2)$. Find $f'(x)$.

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2. Let $y = x^{\log_5(x)}$. Find $\frac{dy}{dx}$.

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3. Use implicit differentiation to find $\frac{dy}{dx}$ for the curve

$$2xy^3 + 3xy = 25.$$

Then find an equation for the tangent line at the point $(5, 1)$.

4. If $\dfrac{1}{x} + \dfrac{1}{y} = 4$ and $y(4) = \dfrac{4}{15}$, find $y'(4)$ by implicit differentiation.

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5. Find an equation of the line tangent to the graph of $(x^2 + y^2)^3 = 8x^2y^2$ at the point $(-1, 1)$.

6. A spherical snowball is melting in such a way that its diameter is decreasing at rate of 0.2 cm/min. At what rate is the volume of the snowball decreasing when the diameter is 17 cm?

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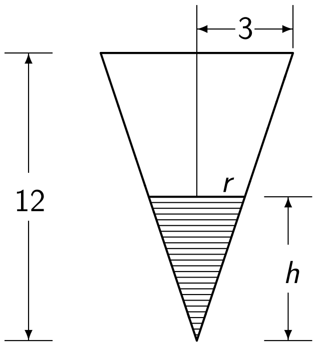
7. A street light is at the top of a 16 ft tall pole. A woman 6 ft tall jogs away from the pole with a speed of 10 ft/sec along a straight path. How fast is the tip of her shadow moving when she is 50 ft from the base of the pole?

8. Water is flowing into a conical tank at 18π cubic feet per hour. The height of the tank is 12 feet and its radius (at the top) is 3 feet. How fast is the depth of water in the tank rising when the water in the tank is 6 feet deep?



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9. Find all critical points of f.

$$f(r) = \frac{2r}{9r^2 + 1}$$

10. Find all critical points of f.

$$f(x) = \sqrt[11]{x}\,(x-1)^2$$

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11. Find the absolute maximum and absolute minimum values of the function

$$f(x) = x^3 + 12x^2 - 27x + 6$$

on each of the indicated intervals. Write *DNE* for any absolute extrema that does not exist.

(A) Interval $= [-10, 0]$.

(B) Interval $= [-7, 2]$.

(C) Interval $= [-10, 2]$.

12. Suppose that

$$f(x) = 3x^4 + 16x^3 + 24x^2 + 7.$$

Show your work for parts (A)-(E) in the space below.

(A) Find all critical points of f.

(B) Give the intervals where $f(x)$ is increasing. Use interval notation.

(C) Give the intervals where $f(x)$ is decreasing.

(D) Find the x-coordinates of all local maxima of f.

(E) Find the x-coordinates of all local minima of f.

Show your work for parts (F)-(H) in the space below.

(F) Use interval notation to indicate where $f(x)$ is concave up.

(G) Use interval notation to indicate where $f(x)$ is concave down.

(H) Find all inflection points of f.

13. Suppose that

$$f(x) = e^x(x^2 - 4x + 1).$$

Show your work for parts (A)-(E) in the space below.

(A) Find all critical points of f.

(B) Give the intervals where $f(x)$ is increasing. Use interval notation.

(C) Give the intervals where $f(x)$ is decreasing.

(D) Find the x-coordinates of all local maxima of f.

(E) Find the x-coordinates of all local minima of f.

Show your work for parts (F)-(H) in the space below.

(F) Use interval notation to indicate where $f(x)$ is concave up.

(G) Use interval notation to indicate where $f(x)$ is concave down.

(H) Find all inflection points of f.

14. Suppose that

$$f(x) = \frac{x^2}{x-1}.$$

Show your work for parts (A)-(E) in the space below.

(A) Find all critical points of f.

(B) Give the intervals where $f(x)$ is increasing. Use interval notation.

(C) Give the intervals where $f(x)$ is decreasing.

(D) Find the x-coordinates of all local maxima of f.

(E) Find the x-coordinates of all local minima of f.

Show your work for parts (F)-(H) in the space below.

(F) Use interval notation to indicate where $f(x)$ is concave up.

(G) Use interval notation to indicate where $f(x)$ is concave down.

(H) Find all inflection points of f.

15. Find the linear approximation of $f(x) = x^5$ at $x = 5$.

Use the linear approximation to estimate 4.9^5.

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16. Find the linear approximation of $f(x) = \sqrt[3]{x}$ at $x = 27$.

Use the linear approximation to estimate $\sqrt[3]{27.4}$.

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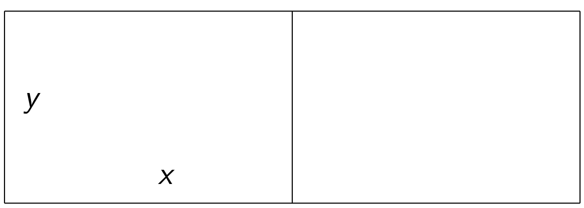
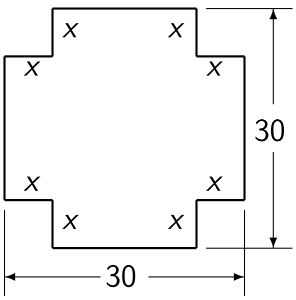
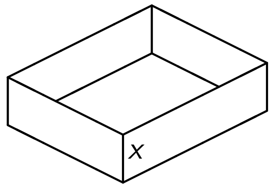
17. A farmer wishes to fence off two identical adjoining rectangular pens, each with 1200 sq ft of area, as shown.



What are x and y so that the least amount of fence is required?

18. A box with an open top is to be formed from a square piece of cardboard which is 30 inches by 30 inches by cutting a square of size x inches from each corner, and folding up the sides. What is the largest possible volume of such a box?





Write the objective function as a function of the single variable x.

Determine the interval of interest.

Show that there is only one local extremum in the interval of interest, and show that it is a local maximum.

How do you know that the local maximum is an absolute maximum?

Determine the maximum volume.

19. Consider the function $f(x) = 2\sqrt{x} + 3$ on the interval $[4, 9]$.

(A) Find the secant line slope on this interval.

(B) By the Mean Value Theorem, we know there exists at least one c in the open interval $(4, 9)$ such that $f'(c)$ is equal to this mean slope. Find all values of c that work.

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20. Evaluate the limit.

$$\lim_{x\to\infty} \left(\frac{10x}{10x+3}\right)^{5x}$$

21. Evaluate the following limits.

(A) $\lim_{x \to 1} \frac{6^x - 6}{x^2 - 1}$

(B) $\lim_{x \to \infty} \frac{\tan^{-1}(x)}{(1/x) - 6}$

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22. Evaluate the following limit using L'Hospital's rule if appropriate.

$\lim_{x \to \infty} \left(\sqrt[3]{x^3 - 7x^2} - x\right)$

23. Suppose f is twice differentiable with

$$f''(x) = 7x - 2\,, \qquad f'(-2) = 0\,, \quad \text{and} \quad f(-2) = -2\,.$$

Find $f'(x)$ and find $f(3)$.

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24. A car traveling at 49 ft/sec decelerates at a constant 4 feet per second squared. How many feet does the car travel before coming to a complete stop?