

Mathematics 1161: Midterm Exam 3 Study Guide

1. **Midterm Exam 3 is on 16 November at 6:00-6:55pm in McPherson Lab 1000.**
It will cover Sections 5.1, 5.2, 5.3, 5.4, 5.5, 6.1, 6.2, 6.3, 6.4, 6.5, 6.6, and 6.7.
2. Take your **BuckID** to the exam. The use of notes, calculators, or other electronic devices is forbidden.
3. Be able to do problems from the quizzes, online homework, and written homework.
4. Save time by memorizing the derivatives of common functions.
5. Several practice exercises appear on the pages that follow. Solutions will be available on Carmen in the Modules section, and we will do a few of these problems during our review session. These exercises primarily reinforce computational skills, but keep in mind that the exam will also test conceptual understanding.
6. **What do you need to know?**
 - 5.1 Know how to estimate a definite integral using left, right, and/or midpoint Riemann sums. (This is in conjunction with the definition of definite integral in Section 5.2.)
 - 5.2 Understand the connection between net area and definite integrals. Know the properties of definite integrals in Table 5.4.
 - 5.3 Know what the Area Function is. Know the Fundamental Theorem of Calculus, and be able to use it to evaluate some definite integrals.
 - 5.4 Know how to find the average value of a function on an interval. Know how the properties of even and odd functions can be used to simplify the evaluation of certain integrals.
 - 5.5 Be able to use the Substitution Rule for Indefinite Integrals and for Definite Integrals.
 - 6.1 Know how to find the velocity function from the acceleration function and how to find the position function from the velocity function. Know what speed, displacement, and distance traveled are.
 - 6.2 Know how to find the area of a region using integrals and horizontal slices or vertical slices.
 - 6.3 Know how to find the volume of a solid using integrals when the base is a given region and the shape of the cross sections (perpendicular to an axis) is given. Be able to use the Disk/Washer Method to find the volume of a solid of revolution.
 - 6.4 Be able to use the Shell Method to find the volume of a solid of revolution.
 - 6.5 Know the arc length formula and be able to use it to find the length of a segment of a curve.
 - 6.6 Know how to find the surface area of a surface of revolution.
 - 6.7 Know how to find the mass of a thin straight wire when given a density function. Know Hooke's Law for a spring and be familiar with spring problems involving force and work. Know how to find the work required to pump a liquid out of a tank to a specified output level.

1. Let $f(x) = x^2 + 1$. Use a right Riemann sum with $n = 6$ to estimate $\int_{-3}^3 f(x) dx$.

2. Compute the derivative below. $\frac{d}{dx} \left[\int_{x^3}^2 \frac{t}{\sqrt{1+t^2}} dt \right]$

3. Let $f(x)$ and $g(x)$ be integrable functions with the properties given below.

$$\int_1^2 f(x) dx = -5$$

$$\int_2^5 g(x) dx = 1$$

$$\int_1^7 f(x) dx = 4$$

$$\int_5^7 g(x) dx = -2$$

Compute $\int_2^7 (2f(x) - g(x) + 1) dx$.

4. Evaluate the definite integral. $\int_{-2}^3 (x^2 - x - 6) dx$

5. Evaluate the definite integral. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\cos x - 1) dx$

6. Evaluate the definite integral. $\int_1^{\sqrt{3}} \frac{dx}{1+x^2}$

7. The height of an arch above the ground is given by the function $y = 12 \sin\left(\frac{\pi x}{4}\right)$ for $0 \leq x \leq 4$. What is the average height of the arch above the ground?

8. Compute the following indefinite integral. $\int \frac{2x^2}{\sqrt{1-4x^3}} dx$

9. Compute the following indefinite integral. $\int \frac{x}{\sqrt{x-4}} dx$

10. Compute the following definite integral. $\int_0^{\frac{\pi}{4}} \frac{\sin x}{\cos^2 x} dx$

11. Compute the following definite integral. $\int_0^{\ln 4} \frac{e^x}{3 + 2e^x} dx$

12. Find the position s and velocity v of an object moving along a straight line with the given acceleration, initial velocity, and initial position.

$$a(t) = e^{-t} \quad v(0) = 6 \quad s(0) = 4$$

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13. A region is bounded by the curves $y = 4x$, $y = -4$, and $y = 12 - 2x$.

- (a) Use vertical slices to express the area of this region using an integral or sum of integrals.
- (b) Use horizontal slices to express the area of this region using an integral or sum of integrals.
- (c) Find the area of this region.

14. A region is bounded by the curves $y = 18 - 3x^2$ and $y = x^2 + 2$.

- (a) Use vertical slices to express the area of this region using an integral or sum of integrals.
- (b) Use horizontal slices to express the area of this region using an integral or sum of integrals.
- (c) Find the area of this region.

15. The base of a solid is the region in the xy -plane bounded by $y = 4x$, $y = 2x + 8$, and $x = 0$. The cross sections through the solid perpendicular to the x -axis are squares. Find the volume of the solid.

16. The region \mathcal{R} is bounded by the curves $y = 10 - 3x^3$, $y = 1$, $x = 0$ and $x = 1$.

(a) A solid is formed by revolving \mathcal{R} about the x -axis.

- i. Use the Disk/Washer Method to express the volume of this solid as an integral or sum of integrals.
- ii. Use the Shell Method to express the volume of this solid as an integral or sum of integrals.
- iii. Find the volume of this solid.

(b) A solid is formed by revolving \mathcal{R} about the y -axis.

- i. Use the Disk/Washer Method to express the volume of this solid as an integral or sum of integrals.
- ii. Use the Shell Method to express the volume of this solid as an integral or sum of integrals.
- iii. Find the volume of this solid.

17. The region \mathcal{R} is bounded by the curves $y = \sqrt{x}$, $y = 3 - 2x$, and $y = 0$.

(a) A solid is formed by revolving \mathcal{R} about the x -axis.

- i. Use the Disk/Washer Method to express the volume of this solid as an integral or sum of integrals.
- ii. Use the Shell Method to express the volume of this solid as an integral or sum of integrals.
- iii. Find the volume of this solid.

(b) A solid is formed by revolving \mathcal{R} about the y -axis.

- i. Use the Disk/Washer Method to express the volume of this solid as an integral or sum of integrals.
- ii. Use the Shell Method to express the volume of this solid as an integral or sum of integrals.
- iii. Find the volume of this solid.

18. Find the length of the segment of the curve $y = 1 + 5x^{\frac{3}{2}}$ from $x = 0$ to $x = 4$.

19. Find the length of the segment of the curve $x = \frac{3}{4}y^4 + \frac{1}{24}y^{-2}$ from $y = 1$ to $y = 2$.

20. A surface of revolution is formed by revolving the curve $y = 2x^3$ from $x = 0$ to $x = 1$ about the x -axis. Find the surface area.

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21. A surface of revolution is formed by revolving the curve $x = \sqrt{2y + 1}$ from $x = 1$ to $x = 3$ about the y -axis. Find the surface area.

22. A thin rod from $x = 1$ to $x = b$ (in m) has density $\rho(x) = x$ (in kg/m).

(a) What is the mass of the rod in terms of b .

(b) If the rod has a mass of 40 kg, what is b ? How long is the rod?

23. A spring with spring constant k requires 80 N to hold in a stretched position 2 m past its equilibrium position.

(a) How much work was required to stretch the spring?

(b) How much additional work is required to stretch the spring an additional 2 m?

24. A tank is formed by revolving the parabolic segment $y = 3x^2$ from $x = 0$ to $x = 2$ (in m) about the y -axis. The tank is filled to a depth of 10 m with water ($\rho = 1000 \text{ kg/m}^3$). How much work is required to pump all the water to the level of the top of the tank and out of the tank?