Lecture 11 – September 17, 2018

- Midterm 1 Fri. 9/21 during class – covers through 3.7 but not 3.8
  - Practice for Midterm 1 & Study guide solutions posted `https://people.math.osu.edu/broaddus.9/11610X/`
  - Study materials
    - practice midterm 1
    - study guide 1
    - previous quizzes
    - homework
    - lecture notes
    - textbook
  - Human resources
    - My office hours – M & W 12:30pm-2pm in Math Tower (MW) 650
    - Recitation Instructor
    - MSLC – `https://mslc.osu.edu/courses/math/1161`
- Written Homework 2 due in recitation on Tues. 9/18

Today

- 1-dimensional motion
- Population growth
- Derivatives in economics

Derivatives in motion

How can we use math to predict the motion of objects (say in free-fall). Let's develop a model. First let's name some salient quantities:

Definition (Quantities related to motion)

If the **position** of an object at time t is $s(t)$ then

- Let Δt be then change in time
- The **displacement** from time t to $t + \Delta t$ is $\Delta s = s(t + \Delta t) - s(t)$.
- The **average velocity** from time t to $t + \Delta t$ is $v_{av} = \frac{\Delta s}{\Delta t} = \frac{s(t+\Delta t)-s(t)}{\Delta t}$
- The **instantaneous velocity** (or just **velocity**) at time t is $v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \lim_{\Delta t \to 0} \frac{s(t+\Delta t)-s(t)}{\Delta t} = s'(t)$
- The **speed** is $|v| = |s'(t)|$
- The **acceleration** at time t is $a(t) = \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}^2 s} t^2 = s''(t)$

Example (Free fall)

Suppose that the height of an object at time t (in seconds) has position function

$$s(t) = -4.9t^2 + 2t + 5 \quad \text{(meters)}$$

1. Find the velocity and acceleration
   $v(t) = -9.8t + 2$ (meters/second)
   $a(t) = -9.8$ (meters^2/second)
2. Highest point?
   $0 = -9.8t + 2$ so $t = \frac{2}{9.8} = \frac{10}{49}$ seconds,
   $s(\frac{10}{49}) = -4.9 \cdot (\frac{10}{49})^2 + 2(\frac{10}{49}) + 5$ meters ≈ 5.20408 meters
3. What will be the velocity when the rock hits the ground?
   $0 = -4.9t^2 + 2t + 5$ so $t = \frac{-2 \pm \sqrt{2^2 - 4(-4.9)(5)}}{2(-4.9)} = \frac{-2 \pm \sqrt{102}}{-9.8}$ seconds
   positive root is $t = \frac{-2-\sqrt{102}}{-9.8}$ seconds ≈ 1.2346 seconds
   $v = -9.8(\frac{-2-\sqrt{102}}{-9.8}) + 2 = -\sqrt{102}$ meters/second $\approx$ -10.0995 meters/second

Population Growth

Definition

Suppose the size of some quantitity (population, price, etc.) at time t is $p(t)$.

- Let Δt be then change in time
- the **growth** from time t to $t + \Delta t$ is $\Delta p = p(t + \Delta t) - p(t)$.
- The **average growth rate** from time t to $t + \Delta t$ is $g_{\text{ave}} = \frac{\Delta p}{\Delta t} = \frac{p(t+\Delta t)-p(t)}{\Delta t}$
- The **instantaneous growth rate** (or just **growth rate**) at time t is $g_{\text{inst}} = \frac{\mathrm{d}p}{\mathrm{d}t} = \lim_{\Delta t \to 0} \frac{\Delta p}{\Delta t} = \lim_{\Delta t \to 0} \frac{p(t+\Delta t)-p(t)}{\Delta t} = p'(t)$

Example (Petri dish)

Suppose that the population of bacteria in a petri dish at time t (in days) is

$$p(t) = 1000e^t$$

1. What is the average growth rate from time $t = 2$ to $t = 8$?
   $g_{\text{ave}} = \frac{1000e^8 - 1000e^2}{8-2}$ bacteria/day ≈ 495594 bacteria/day
2. Instantaneous growth rate at $t = 6$?
   $g_{\text{inst}} = 1000e^6$ bacteria/day ≈ 403428 bacteria/day

Derivatives in economics

Definition

Suppose the cost of producing x items is given by the **cost function** $C(x)$

- The **average cost** to produce x items is

$$\overline{C}(x) = \frac{C(x)}{x}$$

- The **marginal cost**

$$C'(x)$$

  is the cost to produce one more item after producing x items.

Example (Printing textbooks)

Suppose that the cost of printing x textbooks is

$$C(x) = -0.03x^2 + 10x + 500$$

1. What is the average cost of printing 100 textbooks?
   $\overline{C}(x) = \frac{-0.03 \cdot 100^2 + 10 \cdot 100 + 500}{100} = 12$
2. Marginal cost of printing 100th textbook?
   $C'(x) = -0.06 \cdot 100 + 10 = 4$

More derivatives in economics

Definition

Suppose that the price of a good is p

- The **demand** $D(p)$ at the price p is the number of units of the good you can sell at price a price of p
- For a change in price from p to $p + \Delta p$ the change in demand is $\Delta D = D(p + \Delta p) - D(p)$
- The **elasticity** of the good at the price p is

$$E(p) = \lim_{\Delta p \to 0} \frac{\left(\frac{\Delta D}{D}\right)}{\left(\frac{\Delta p}{p}\right)} = \frac{\mathrm{d}D}{\mathrm{d}p} \frac{p}{D(p)}$$

- If $-\infty < E(p) < -1$ then the good is **elastic** (increasing price decreases revenue) at price p.
- If $-1 < E(p) < 0$ then the good is **inelastic** (increasing price brings in more revenue) at price p.

Example (Pricing computers)

The demand functions for imported computers is $D(p) = 1000(1000 - p)$.

1. What is the elasticity of this computer at a price of 100?
   $E(100) = \left(\frac{\mathrm{d}D}{\mathrm{d}p}\Big|_{p=100}\right) \frac{100}{1000(1000-100)} = \frac{-1000 \cdot 100}{1000(900)} = -\frac{1}{9}$
2. Should the importer worry about passing a 10% import tariff on the the consumer if the price is currently 100?
   No. At a price of 100 this product is inelastic.
3. At what price would this good become elastic?

$$\begin{aligned} -1 &= E(p) \\ -1 &= -1000 \cdot \frac{p}{1000(1000-p)} \\ 1000 - p &= p \\ 1000 &= 2p \\ 500 &= p \end{aligned}$$