Lecture 17 – October 3, 2018

- OH 4.4 due tomorrow (10/4)
- OH 4.5,4.6 due Tues. 10/9

Today

- Complete graphing

Complete graphing

Graphing

1. Domain
2. Horizontal and vertical asymptotes
3. Symmetry (odd? even? periodic?)
4. Compute f' and f''
5. Critical points, Inflection point (candidates)
6. Intervals of increase/decrease and concavity
7. Evaluate to get local extrema and inflection points
8. Intercepts x-intercepts (set $y = 0$). y-intercept (plug in $x = 0$)
9. Choose graphing window (include all info from above)

Example

Graph $f(x) = (x-3)(x^2+12x+90) + 150 \ln|x-3|$

1. Domain: $|x-3| > 0$ if $x-3>0$ or $x-3<0$ so domain is $(-\infty, 3) \cup (3, \infty)$
2. Asymptotes:

$$\lim_{x\to 3} f(x) = \lim_{x\to 3}(x-3)(x^2+12x+90) + 150\ln|x-3| = -\infty$$
$$\lim_{x\to \infty} f(x) = \lim_{x\to \infty}(x-3)(x^2+12x+90) + 150\ln|x-3| = \infty$$
$$\lim_{x\to -\infty} f(x) = \lim_{x\to \infty}(x-3)(x^2+12x+90) + 150\ln|x-3| = -\infty$$

One vertical asymptote: $x = 3$
No horizontal asymptotes

Example (*continued*)

3. Symmetry:

$$f(4) = (4-3)(4^2 + 12\cdot 4 + 90) + 150\ln|4-3| = 154$$

$$f(-4) = (-4-3)((-4)^2 + 12\cdot(-4) + 90) + 150\ln|-4-3| \approx -114.11$$

   Thus f is not odd, even or periodic.

4. f' and f'':

$$\begin{aligned} f'(x) &= x^2 + 12x + 90 + (x-3)(2x+12) + \frac{150}{x-3} \\ &= x^2 + 12x + 90 + 2x^2 + 12x - 6x - 36 + \frac{150}{x-3} \\ &= 3x^2 + 18x + 54 + \frac{150}{x-3} \\ &= \frac{(3x^2+18x+54)(x-3)+150}{x-3} \\ &= \frac{3x^3 + 18x^2 + 54x - 9x^2 - 54x - 162 + 150}{x-3} \end{aligned}$$

Example (*continued*)

4. *(continued)*

$$\begin{aligned} &= \frac{3x^3 + 9x^2 - 12}{x-3} \\ &= \frac{3(x^3 + 3x^2 - 4)}{x-3} \\ &= \frac{3(x-1)(x+2)^2}{x-3} \end{aligned}$$

$$\begin{aligned} f''(x) &= \frac{(9x^2+18x)(x-3) - (3x^3+9x^2-12)\cdot 1}{(x-3)^2} \\ &= \frac{9x^3 + 18x^2 - 27x^2 - 54x - 3x^3 - 9x^2 + 12}{(x-3)^2} \end{aligned}$$

Example (*continued*)

4. *(continued)*

$$= \frac{6x^3 - 18x^2 - 54x + 12}{(x-3)^2}$$
$$= \frac{6(x^3 - 3x^2 - 9x + 2)}{(x-3)^2}$$
$$= \frac{6(x+2)(x^2 - 5x + 1)}{(x-3)^2}$$

5. Intervals of Increase: $(-\infty, -2)$ and $(-2, 1)$ SO $(-\infty, 1]$ and $(3, \infty)$
   Intervals of Decrease: $[1, 3)$
   Concave Down: $(-\infty, -2]$, $[\frac{5-\sqrt{21}}{2}, 3)$, $(3, \frac{5+\sqrt{21}}{2}]$
   Concave Up: $[-2, \frac{5-\sqrt{21}}{2}]$, $[\frac{5+\sqrt{21}}{2}, \infty)$
6. Critical Points: $x = -2$, $x = 1$
   Inflection Points: at $x = -2$, $x = \frac{5-\sqrt{21}}{2} \approx 0.21$, $x = \frac{5+\sqrt{21}}{2} \approx 4.79$

Example (*continued*)

7. Local max at $x = 1$ so we have a local maximum of

$$\begin{aligned} f(1) &= (1-3)(1^2 + 12 \cdot 1 + 90) + 150 \ln|1-3| \\ &= -2(1 + 12 + 90) + 150 \ln 2 \\ &= -206 + 150 \ln 2 \\ &\approx -102.03 \end{aligned}$$

Inflection point at $x = -2$

$$\begin{aligned} f(-2) &= (-2-3)((-2)^2 + 12 \cdot (-2) + 90) + 150 \ln|-2-3| \\ &= -350 + 150 \ln 5 \\ &\approx -108.58 \end{aligned}$$

Inflection point at $x = \frac{5-\sqrt{21}}{2}$

$$\begin{aligned} f(\tfrac{5-\sqrt{21}}{2}) &= (\tfrac{5-\sqrt{21}}{2} - 3)((\tfrac{5-\sqrt{21}}{2})^2 + 12 \cdot (\tfrac{5-\sqrt{21}}{2}) + 90) + 150 \ln|\tfrac{5-\sqrt{21}}{2} - 3| \\ &\approx -104.34 \end{aligned}$$

Example (*continued*)

7. *(continued)*

   Inflection point at $x = \frac{5+\sqrt{21}}{2}$

   $$f(\tfrac{5+\sqrt{21}}{2}) = (\tfrac{5+\sqrt{21}}{2} - 3)((\tfrac{5+\sqrt{21}}{2})^2 + 12 \cdot (\tfrac{5+\sqrt{21}}{2}) + 90) + 150 \ln |\tfrac{5+\sqrt{21}}{2} - 3|$$
   $$\approx 392.77$$

8. x-intercepts

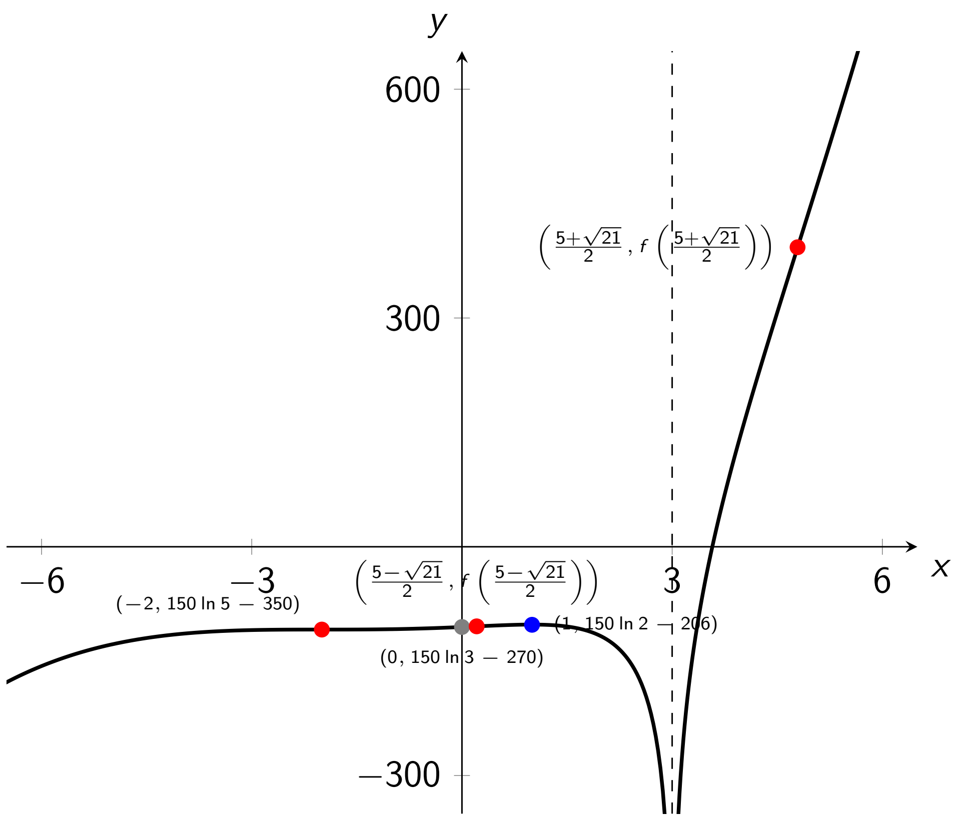
   $$0 = (x-3)(x^2 + 12x + 90) + 150 \ln |x - 3| \quad \text{not solvable}$$

   y-intercept:

   $$\begin{aligned} y &= (0-3)(0^2 + 12 \cdot 0 + 90) + 150 \ln |0 - 3| \\ &= -270 + 150 \ln 3 \\ &\approx -105.2 \end{aligned}$$

Example (*continued*)

9. Plot



Example

Graph the function $g(x) = 1 + \frac{1}{x} + \frac{1}{x^2} = 1 + x^{-1} + x^{-2}$

1. Domain: $x \neq 0$ so domain is $(-\infty, 0) \cup (0, \infty)$
2. Asymptotes:

$$\lim_{x\to 0} g(x) = \lim_{x\to 0} \frac{x^2+x+1}{x^2} = \infty$$
$$\lim_{x\to \infty} g(x) = \lim_{x\to \infty} 1 + \frac{1}{x} + \frac{1}{x^2} = 1$$
$$\lim_{x\to -\infty} g(x) = \lim_{x\to -\infty} 1 + \frac{1}{x} + \frac{1}{x^2} = 1$$

   One vertical asymptote: $x = 0$
   One horizontal asymptote: $y = 1$

Example (*continued*)

3. Symmetry:

$$g(1) = 1 + \tfrac{1}{1} + \tfrac{1}{1^2} = 3$$
$$g(-1) = 1 + \tfrac{1}{-1} + \tfrac{1}{(-1)^2} = 1$$

   Thus g is not odd, even or periodic.

4. g' and g'':

$$g'(x) = (-1)x^{-2} - 2x^{-3}$$
$$= -\frac{x+2}{x^3}$$

$$g''(x) = 2x^{-3} + 6x^{-4}$$
$$= \frac{2x+6}{x^4}$$

Example (*continued*)

5. Critical Point: $x = -2$
   Inflection Point: at $x = -3$
6. Intervals of Increase: $[-2, 0)$
   Intervals of Decrease: $(-\infty, -2]$ and $(0, \infty)$
   Concave Down: $(-\infty, -3]$
   Concave Up: $[-3, 0)$ and $(0, \infty)$
7. Local min at $x = -2$

$$\begin{aligned} g(-2) &= 1 - \tfrac{1}{2} + \tfrac{1}{4} \\ &= \tfrac{3}{4} \end{aligned}$$

   Inflection point at $x = -3$

$$\begin{aligned} g(-3) &= 1 - \tfrac{1}{3} + \tfrac{1}{9} \\ &= \tfrac{7}{9} \end{aligned}$$

Example (*continued*)

8. x-intercepts

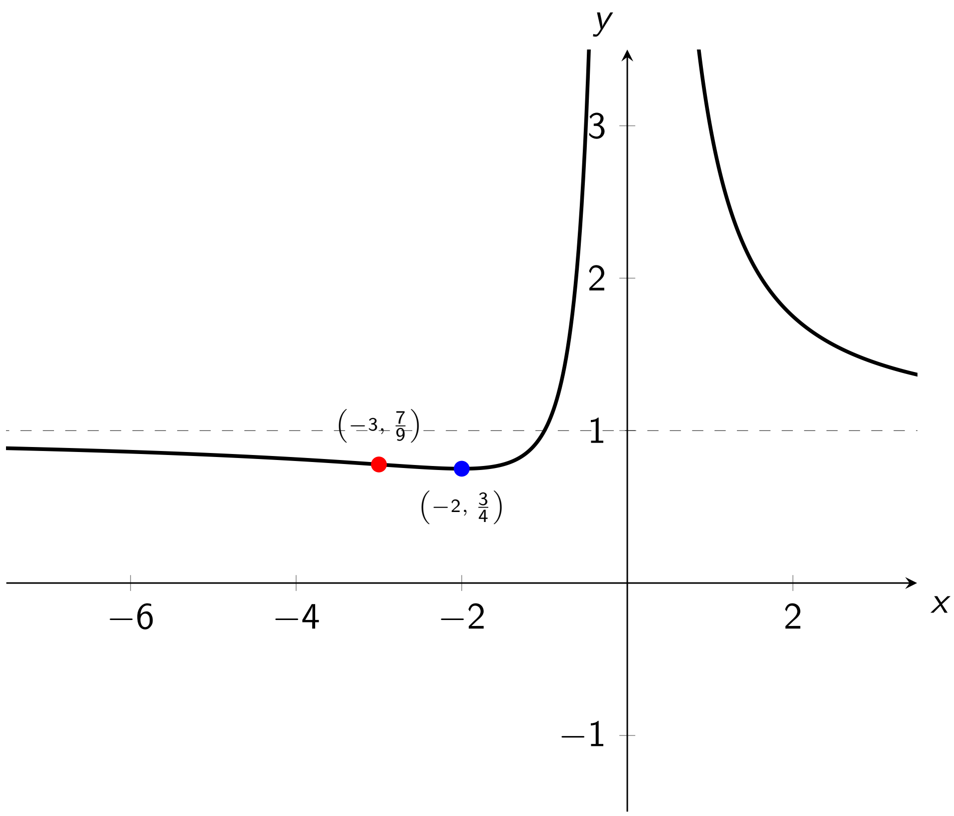
$$0 = \frac{x^2 + x + 1}{x^2}$$
$$0 = x^2 + x + 1$$

   no solution so no x-intercept

   y-intercept: 0 not in domain of g so no y-intercept

Example (*continued*)

9. Plot



Example

Graph the function $h(x) = x\sqrt{9 - x^2}$

1. Domain: $9 - x^2 \geq 0$ so $x^2 \leq 9$ so $|x| \leq 3$ domain is $[-3, 3]$
2. Asymptotes:
   No vertical asymptotes since h is continuous on its domain.
   No horizontal asymptotes since domain is bounded.
3. Symmetry:

$$\begin{aligned} h(-x) &= (-x)\sqrt{9 - (-x)^2} \\ &= -x\sqrt{9 - x^2} \\ &= -h(x) \end{aligned}$$

   Thus h is odd.
   h is not even or periodic.

Example (*continued*)

4. h' and h'':

$$\begin{aligned} h'(x) &= (9-x^2)^{1/2} + x \cdot \tfrac{1}{2}(9-x^2)^{-1/2}(-2x) \\ &= \frac{9-x^2}{(9-x^2)^{1/2}} - \frac{x^2}{(9-x^2)^{1/2}} \\ &= \frac{9-2x^2}{(9-x^2)^{1/2}} \end{aligned}$$

$$\begin{aligned} h''(x) &= \frac{-4x(9-x^2)^{1/2} - (9-2x^2)\cdot \frac{1}{2} \cdot (9-x^2)^{-1/2}(-2x)}{9-x^2} \\ &= \frac{-4x(9-x^2) + (9-2x^2)x}{(9-x^2)^{3/2}} \\ &= \frac{x(-36+4x^2+9-2x^2)}{(9-x^2)^{3/2}} \\ &= \frac{x(2x^2-27)}{(9-x^2)^{3/2}} \end{aligned}$$

Example (*continued*)

5. Critical Points: $x = -\frac{3}{\sqrt{2}}$ and $x = \frac{3}{\sqrt{2}}$
   Inflection Point (candidates): at $x = -\sqrt{\frac{27}{2}}$ (outside domain), $x = \sqrt{\frac{27}{2}}$ (outside domain) and $x = 0$
6. Intervals of Increase: $[-\frac{3}{\sqrt{2}}, \frac{3}{\sqrt{2}}]$
   Intervals of Decrease: $[-3, \frac{3}{\sqrt{2}}]$ and $[-\frac{3}{\sqrt{2}}, 3]$
   Concave Down: $[0, 3]$
   Concave Up: $[-3, 0]$

Example (*continued*)

6. Local min at $x = -\frac{3}{\sqrt{2}}$

$$f(-\tfrac{3}{\sqrt{2}}) = -\tfrac{3}{\sqrt{2}}\sqrt{9-\left(-\tfrac{3}{\sqrt{2}}\right)^2}$$
$$= -\tfrac{9}{2}$$

Local max at $x = \frac{3}{\sqrt{2}}$

$$f(\tfrac{3}{\sqrt{2}}) = \tfrac{3}{\sqrt{2}}\sqrt{9-\left(\tfrac{3}{\sqrt{2}}\right)^2}$$
$$= \tfrac{9}{2}$$

7. x-intercepts

$$0 = x\sqrt{9-x^2}$$

so

$$x = 0 \quad \text{OR} \qquad\qquad 9 - x^2 = 0$$

x-intercepts at $x = 0$, $x = -3$ or $x = 3$ no solution so no x-intercept

y-intercept: $h(0) = 0\sqrt{9-0^2} = 0$ so $y = 0$

Example (*continued*)

9. Plot