1. Let $f(x) = x^2 \ln(9 - 5x^2)$. Find $f'(x)$.

$$f'(x) = 2x \ln(9-5x^2) + x^2 \cdot \frac{1}{9-5x^2} \frac{d}{dx}[9-5x^2]$$

$$= 2x \ln(9-5x^2) + x^2 \cdot \frac{1}{9-5x^2} \cdot (-10x)$$

$$= 2x \ln(9-5x^2) - \frac{10x^3}{9-5x^2}$$

---

2. Let $y = x^{\log_5(x)}$. Find $\frac{dy}{dx}$.

Method 1.

$$\ln y = \log_5 x \ln x$$

$$\frac{1}{y} \cdot y' = \frac{1}{x \ln 5} \cdot \ln x + \log_5 x \cdot \frac{1}{x}$$

$$\frac{dy}{dx} = y' = x^{\log_5(x)} \left[\frac{\ln x}{x \ln 5} + \frac{\log_5 x}{x} \right]$$

Note: Using Change of Base Formula,

$$\frac{dy}{dx} = x^{\log_5 x} \cdot \frac{2\log_5 x}{x}$$

Method 2:

$$y = (e^{\ln x})^{\log_5 x}$$

$$= e^{\ln x \cdot \log_5 x}$$

$$\frac{dy}{dx} = e^{\ln x \cdot \log_5 x} \left(\frac{1}{x} \cdot \log_5 x + \ln x \cdot \frac{1}{x \ln 5} \right)$$

$$= x^{\log_5 x} \left(\frac{\log_5 x}{x} + \frac{\ln x}{x \ln 5} \right)$$

---

3. Use implicit differentiation to find $\frac{dy}{dx}$ for the curve

$$2xy^3 + 3xy = 25.$$

Then find an equation for the tangent line at the point $(5, 1)$.

$$2y^3 + 2x(3y^2 y') + 3y + 3xy' = 0$$

$$6xy^2 y' + 3xy' = -2y^3 - 3y$$

$$y'(6xy^2 + 3x) = -2y^3 - 3y$$

$$\boxed{y' = \frac{-2y^3 - 3y}{6xy^2 + 3x}}$$

$$m = y'\big|_{(5,1)} = \frac{-2-3}{30+15} = -\frac{5}{45}$$

$$= -\frac{1}{9}$$

$$\boxed{y - 1 = -\frac{1}{9}(x - 5)}$$

4. If $\frac{1}{x} + \frac{1}{y} = 4$ and $y(4) = \frac{4}{15}$, find $y'(4)$ by implicit differentiation.

$$-\frac{1}{x^2} - \frac{1}{y^2}y' = 0$$

$$-\frac{1}{y^2}y' = \frac{1}{x^2}$$

$$y' = -\frac{y^2}{x^2}$$

$$y'(4) = -\frac{\left(\frac{4}{15}\right)^2}{4^2} = -\frac{4^2}{15^2}\cdot\frac{1}{4^2} = -\frac{1}{15^2} = -\frac{1}{225}$$

---

5. Find an equation of the line tangent to the graph of $(x^2 + y^2)^3 = 8x^2y^2$ at the point $(-1, 1)$.

Take $\frac{d}{dx}$ of both sides:

$$3(x^2+y^2)^2(2x+2yy') = 16xy^2 + 16x^2yy'$$

We are not asked to solve for y' in general, so we may substitute for x and y:

$$3(1+1)^2(-2+2y') = -16 + 16y'$$

$$-24 + 24y' = -16 + 16y'$$

$$8y' = 8$$

$$y' = 1$$

$$m = y'\big|_{(-1,1)} = 1$$

$$y - 1 = (1)(x - (-1))$$

$$\boxed{y - 1 = x + 1}$$

6. A spherical snowball is melting in such a way that its diameter is decreasing at rate of 0.2 cm/min. At what rate is the volume of the snowball decreasing when the diameter is 17 cm?

V= volume of snowball

r= radius of snowball

D= diameter of snowball

$V = \frac{4}{3}\pi r^3$

$D = 2r$

$r = \frac{1}{2}D$

$V = \frac{4}{3}\pi(\frac{1}{2}D)^3$

$V = \frac{4}{3}\pi \frac{1}{8} D^3$

$V = \frac{\pi}{6} D^3$

Take $\frac{d}{dt}$ of both sides:

$\frac{dV}{dt} = \frac{\pi}{6} \cdot 3D^2 \frac{dD}{dt}$

$\frac{dV}{dt} = \frac{\pi}{2}(17)^2(-0.2)$

$= -28.9\pi \ \text{cm}^3/\text{min}$

Given: $\frac{dD}{dt} = -.2$ cm/min

Want: $\frac{dV}{dt}$ @ D=17 cm

The volume is decreasing at a rate of 28.9π cm^3/min.

---

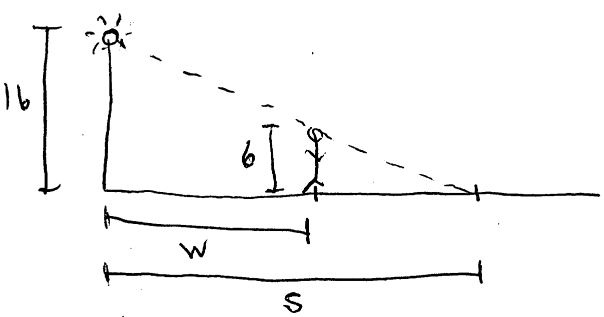
7. A street light is at the top of a 16 ft tall pole. A woman 6 ft tall jogs away from the pole with a speed of 10 ft/sec along a straight path. How fast is the tip of her shadow moving when she is 50 ft from the base of the pole?

w = distance between pole and woman

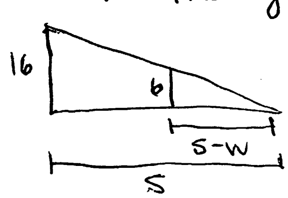
s = distance between pole and tip of shadow

Given: $\frac{dw}{dt} = 10$ ft/s

Want: $\frac{ds}{dt}$ @ w=50 ft.



Similar triangles:



$\frac{16}{s} = \frac{6}{s-w}$

$16(s-w) = 6s$

$16s - 16w = 6s$

$10s = 16w$

$s = \frac{8}{5}w$

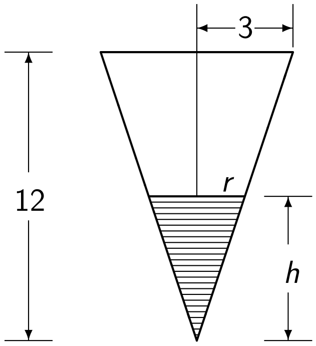
$\frac{ds}{dt} = \frac{8}{5} \cdot \frac{dw}{dt}$

$\frac{ds}{dt} = \frac{8}{5}(10)$

$\boxed{\frac{ds}{dt} = 16 \ \text{ft/s}}$

Note: We did not need the @ info for this particular problem.

8. Water is flowing into a conical tank at 18π cubic feet per hour. The height of the tank is 12 feet and its radius (at the top) is 3 feet. How fast is the depth of water in the tank rising when the water in the tank is 6 feet deep?

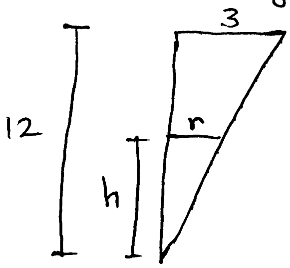


V = volume of water in the tank

$V = \frac{1}{3}\pi r^2 h$

Given: $\frac{dV}{dt} = 18\pi$ ft³/hr Want: $\frac{dh}{dt}$ @ $h = 6$ ft.

Similar triangles:



$$\frac{r}{h} = \frac{3}{12}$$

$$r = \frac{1}{4}h$$

$$V = \frac{\pi}{3} r^2 h$$
$$= \frac{\pi}{3}\left(\frac{1}{4}h\right)^2 h$$
$$= \frac{\pi}{48} h^3$$

$$\frac{dV}{dt} = \frac{\pi}{48} \cdot 3h^2 \cdot \frac{dh}{dt}$$

$$\frac{dV}{dt} = \frac{\pi}{16} h^2 \frac{dh}{dt}$$

$$18\pi = \frac{\pi}{16}(6)^2 \frac{dh}{dt}$$

$$\frac{18 \cdot \pi \cdot 16}{\pi \cdot 6 \cdot 6} = \frac{dh}{dt}$$

$$8 \text{ ft/hr} = \frac{dh}{dt}$$

---

9. Find all critical points of f.

$$f(r) = \frac{2r}{9r^2 + 1}$$

$$f'(r) = \frac{(9r^2+1)(2) - (2r)(18r)}{(9r^2+1)^2}$$

denominator never zero, so defined everywhere: no DNE-type

Check for $f' = 0$ type:

set numerator equal to 0

$$(9r^2+1)(2) - (2r)(18r) = 0$$
$$9r^2 + 1 - 18r^2 = 0$$
$$1 = 9r^2$$

$$r^2 = \frac{1}{9}$$

$$\boxed{r = -\frac{1}{3}, \ r = \frac{1}{3}}$$

10. Find all critical points of f.

$$f(x) = \sqrt[11]{x}\,(x-1)^2$$

$f'(x) = \frac{1}{11}x^{-10/11}(x-1)^2 + x^{1/11}(2)(x-1)$

f' undefined at $x=0$ but f is defined at $x=0$, so DNE-type critical point at $x=0$

(And 0 is an interior point of the domain of f)

Check for $f'=0$ types:

$\frac{1}{11x^{10/11}}(x-1)^2 + 2x^{1/11}(x-1) = 0$

$(x-1)^2 + 22x(x-1) = 0$

$x^2 - 2x + 1 + 22x^2 - 22x = 0$

$23x^2 - 24x + 1 = 0$

$(23x-1)(x-1) = 0$

$23x - 1 = 0$ → $x = \frac{1}{23}$

$x - 1 = 0$ → $x = 1$

---

11. Find the absolute maximum and absolute minimum values of the function

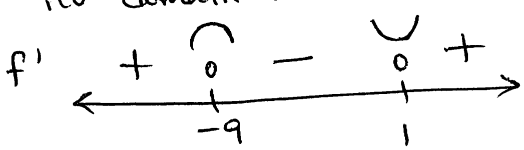
$$f(x) = x^3 + 12x^2 - 27x + 6$$

on each of the indicated intervals. Write *DNE* for any absolute extrema that does not exist.

$f'(x) = 3x^2 + 24x - 27$
$= 3(x^2 + 8x - 9)$
$= 3(x+9)(x-1)$

crit. pts: $x = -9$, $x = 1$

no domain issues



For each interval, we compare the values of f at interior critical points and the endpoints.

$f(-10) = 476$
$f(-9) = 492$
$f(-7) = 440$
$f(0) = 6$
$f(1) = -8$
$f(2) = 8$

(A) Interval $= [-10, 0]$.

For this interval, compare $f(-10)$, $f(-9)$ and $f(0)$.

Abs min: 6 at $x=0$
Abs. max: 492 at $x=-9$

(B) Interval $= [-7, 2]$.

For this interval, compare $f(-7)$, $f(1)$ and $f(2)$.

Abs. min: -8 at $x=1$
Abs. max: 440 at $x=-7$

(C) Interval $= [-10, 2]$.

For this interval, compare $f(-10)$, $f(-9)$, $f(1)$ and $f(2)$.

Abs. min: -8 at $x=1$
Abs max: 492 at $x=-9$

12. Suppose that

$$f(x) = 3x^4 + 16x^3 + 24x^2 + 7.$$

Show your work for parts (A)-(E) in the space below.

$f'(x) = 12x^3 + 48x^2 + 48x$

f' defined everywhere.

No f' DNE type crit. pt.

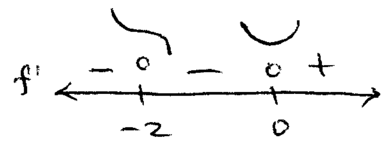
Check for $f' = 0$ type.

$12x^3 + 48x^2 + 48x = 0$

$12x(x^2 + 4x + 4) = 0$

$12x(x+2)^2 = 0$

$x = 0$, $x = -2$



(A) Find all critical points of f.

$x = 0$, $x = -2$

(B) Give the intervals where $f(x)$ is increasing. Use interval notation.

$[0, \infty)$ also acceptable: $(0, \infty)$

(C) Give the intervals where $f(x)$ is decreasing.

$(-\infty, 0]$ also acceptable: $(-\infty, 0)$

Note: near $x=-2$, graph looks like ⌐, which is decreasing.

(D) Find the x-coordinates of all local maxima of f.

none

(E) Find the x-coordinates of all local minima of f.

$x = 0$

Show your work for parts (F)-(H) in the space below.

$f''(x) = 36x^2 + 96x + 48$

no domain issues

$f''(x) = 0 \Leftrightarrow 36x^2 + 96x + 48 = 0$

$12(3x^2 + 8x + 4) = 0$

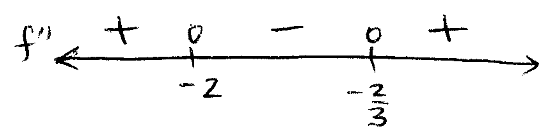
$12(3x+2)(x+2) = 0$

$3x + 2 = 0$

$x = -\frac{2}{3}$

$x + 2 = 0$

$x = -2$



(F) Use interval notation to indicate where $f(x)$ is concave up.

$(-\infty, -2)$, $(-\frac{2}{3}, \infty)$

(G) Use interval notation to indicate where $f(x)$ is concave down.

$(-2, -\frac{2}{3})$

(H) Find all inflection points of f.

f changes concavity and is continuous at $x=-2$ and $x = \frac{-2}{3}$.

So, inflection pts. are at $x=-2$ and $x = -\frac{2}{3}$.

13. Suppose that

$$f(x) = e^x(x^2 - 4x + 1).$$

Show your work for parts (A)-(E) in the space below.

$$f'(x) = e^x(x^2-4x+1) + e^x(2x-4)$$
$$= e^x\left[(x^2-4x+1) + (2x-4)\right]$$
$$= e^x(x^2-2x-3)$$

f' defined everywhere.

no f' DNE type crit. pts.

$f'(x)=0 \Leftrightarrow e^x(x^2-2x-3)=0$

$e^x(x-3)(x+1)=0$

$e^x \neq 0$ b/c $e^x > 0$ $\quad x-3=0$, $x=3$ $\quad x+1=0$, $x=-1$

f': $+$ 0 $-$ 0 $+$ on the number line, with -1 and 3 marked

(A) Find all critical points of f.

$x=-1$, $x=3$

(B) Give the intervals where $f(x)$ is increasing. Use interval notation.

$(-\infty,-1]$, $[3,\infty)$ also acceptable: $(-\infty,-1)$, $(3,\infty)$

(C) Give the intervals where $f(x)$ is decreasing.

$[-1,3]$ also aceptable: $(-1,3)$

(D) Find the x-coordinates of all local maxima of f.

$x=-1$ (by First Derivative Test)

(E) Find the x-coordinates of all local minima of f.

$x=3$ (by First Derivative Test)

Show your work for parts (F)-(H) in the space below.

$$f''(x) = e^x(x^2-2x-3) + e^x(2x-2)$$
$$= e^x\left[(x^2-2x-3) + (2x-2)\right]$$
$$= e^x(x^2-5)$$

no domain issues.

$f''(x)=0 \Leftrightarrow e^x(x^2-5)=0$

$e^x \neq 0$ b/c $e^x>0$ $\quad x^2-5=0$, $x^2=5$, $x=-\sqrt{5}$ or $x=\sqrt{5}$

f'': $+$ 0 $-$ 0 $+$ on the number line, with $-\sqrt{5}$ and $\sqrt{5}$ marked

(F) Use interval notation to indicate where $f(x)$ is concave up.

$(-\infty,-\sqrt{5})$, $(\sqrt{5},\infty)$

(G) Use interval notation to indicate where $f(x)$ is concave down.

$(-\sqrt{5},\sqrt{5})$

(H) Find all inflection points of f.

f changes concavity and is continuous at $x=-\sqrt{5}$ and $x=\sqrt{5}$.

So, inflection points are at $x=-\sqrt{5}$ and $x=\sqrt{5}$.

14. Suppose that

$$f(x) = \frac{x^2}{x-1}.$$

Show your work for parts (A)-(E) in the space below.

$$f'(x) = \frac{(x-1)(2x) - (x^2)(1)}{(x-1)^2} = \frac{x(2x-2-x)}{(x-1)^2} = \frac{x(x-2)}{(x-1)^2}$$

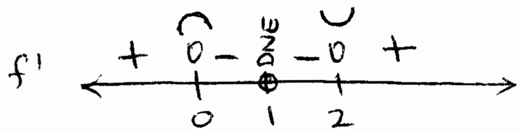
no f' DNE type crit. pts.

$f'(x) = 0 \iff$ numerator $= 0$

$x(x-2) = 0$

$x = 0 \quad x = 2$

f and f' undefined at $x = 1$; no new domain issues.



Note: remember to label domain issues on your number line.

(A) Find all critical points of f.

$x = 0$, $x = 2$

(B) Give the intervals where $f(x)$ is increasing. Use interval notation.

$(-\infty, 0]$, $[2, \infty)$ also acceptable: $(-\infty, 0)$, $(2, \infty)$

(C) Give the intervals where $f(x)$ is decreasing.

$[0, 1)$, $(1, 2]$ also acceptable: $(0, 1)$, $(1, 2)$

(D) Find the x-coordinates of all local maxima of f.

$x = 0$

(E) Find the x-coordinates of all local minima of f.

$x = 2$

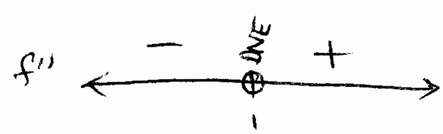
Show your work for parts (F)-(H) in the space below.

$$f'(x) = \frac{x^2 - 2x}{(x-1)^2}$$

$$f''(x) = \frac{(x-1)^2(2x-2) - (x^2-2x)\cdot 2(x-1)(1)}{(x-1)^4} = \frac{(x-1)(2x-2) - 2(x^2-2x)}{(x-1)^3} = \frac{2x^2 - 4x + 2 - 2x^2 + 4x}{(x-1)^3}$$

$$f''(x) = \frac{2}{(x-1)^3}$$

no new domain issues; $f''(x) \neq 0$ for all $x \neq 1$.



(F) Use interval notation to indicate where $f(x)$ is concave up.

$(1, \infty)$

(G) Use interval notation to indicate where $f(x)$ is concave down.

$(-\infty, 1)$

(H) Find all inflection points of f.

f changes concavity at $x = 1$ but f is not continuous at $x = 1$.

So, none. (There is a VA at $x = 1$.)

15. Find the linear approximation of $f(x) = x^5$ at $x = 5$.

$f(a) = f(5) = 5^5 = 3125$

$f'(x) = 5x^4$

$f'(a) = f'(5) = 5(5)^4 = 3125$

$L(x) = f(a) + f'(a)(x-a) = 3125 + 3125(x-5)$

Use the linear approximation to estimate 4.9^5.

$$4.9^5 = f(4.9) \approx L(4.9) = 3125 + 3125(4.9-5) = 3125 + 3125(-0.1) = 3125 - 312.5 = 2812.5$$

---

16. Find the linear approximation of $f(x) = \sqrt[3]{x}$ at $x = 27$.

$f(a) = f(27) = \sqrt[3]{27} = 3$

$f'(x) = \frac{1}{3}x^{-2/3} = \frac{1}{3(\sqrt[3]{x})^2}$

$f'(a) = f'(27) = \frac{1}{3(\sqrt[3]{27})^2} = \frac{1}{27}$

$L(x) = f(a) + f'(a)(x-a) = 3 + \frac{1}{27}(x-27)$

Use the linear approximation to estimate $\sqrt[3]{27.4}$.

$$\sqrt[3]{27.4} = f(27.4) \approx L(27.4) = 3 + \frac{1}{27}(27.4-27) = 3 + \frac{1}{27}(0.4) = 3 + \frac{1}{27}\left(\frac{2}{5}\right) = 3 + \frac{2}{135}$$

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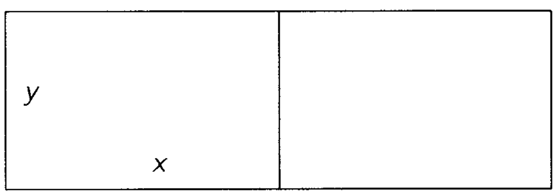
17. A farmer wishes to fence off two identical adjoining rectangular pens, each with 1200 sq ft of area, as shown.



$x = 30$ $\quad y = \frac{1200}{30} = 40$

$x = 30$ ft
$y = 40$ ft

What are x and y so that the least amount of fence is required?

Objective function: F = amount of fence

$F = 4x + 3y$

Constraint: $xy = 1200$

$y = \frac{1200}{x}$

$F = 4x + 3\left(\frac{1200}{x}\right)$

$F(x) = 4x + \frac{3600}{x}$

x and y are lengths w/ $xy = 1200$, so $x > 0$ and $y > 0$. Gives interval of interest $(0, \infty)$.

Crit. pts.?

$F'(x) = 4 - \frac{3600}{x^2}$

no new domain issues

$F'(x) = 0 \Leftrightarrow 4 - \frac{3600}{x^2} = 0$

$4 = \frac{3600}{x^2}$

$x^2 = 900$

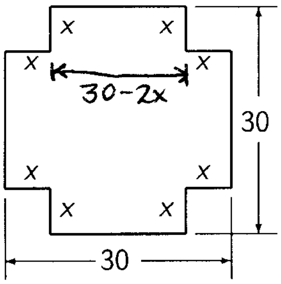
$x = -30$ or $x = 30$

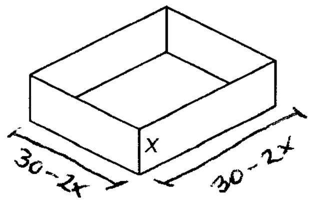
Discard $x = -30$ b/c it's not in the interval of interest.

F': $-$ on $(0, 30)$, 0 at 30, $+$ after. F has local min. at $x = 30$.

Only one local extremum in the interval of interest, so F has absolute minimum at $x = 30$.

18. A box with an open top is to be formed from a square piece of cardboard which is 30 inches by 30 inches by cutting a square of size x inches from each corner, and folding up the sides. What is the largest possible volume of such a box?





length = $30-2x$
width = $30-2x$
height = x

Write the objective function as a function of the single variable x.

$$V(x) = (30-2x)(30-2x)x = x(30-2x)^2$$

Determine the interval of interest. All box dimensions are nonnegative.

$0 \le x$ and $0 \le 30-2x$, $2x \le 30$, $x \le 15$

Interval of interest: $[0, 15]$ also acceptable: $(0, 15)$

Show that there is only one local extremum in the interval of interest, and show that it is a local maximum.

$$V(x) = x(30-2x)^2$$
$$V'(x) = (30-2x)^2 + x \cdot 2(30-2x)(-2)$$
$$= (30-2x)(30-2x-4x)$$
$$= (30-2x)(30-6x)$$

No V' DNE type crit. pts.

$V'(x) = 0$
$(30-2x)(30-6x) = 0$
$30-2x=0$, $30=2x$, $x=15$ ← not an interior point of interval of interest
$30-6x=0$, $30=6x$, $x=5$

V′: + 0 − on $[0, 15]$ with 0 at 5

local max. at $x=5$ by 1st Deriv. Test

How do you know that the local maximum is an absolute maximum?

Only one local extremum on interval of interest, so the local max is also an absolute max.

Determine the maximum volume.

$$V(5) = 5(30-2(5))^2 = 5 \cdot 20^2 = 2000 \text{ in}^3$$

19. Consider the function $f(x) = 2\sqrt{x} + 3$ on the interval $[4, 9]$.

(A) Find the secant line slope on this interval.

$$f(4) = 2\sqrt{4} + 3 = 7$$
$$f(9) = 2\sqrt{9} + 3 = 9$$

$$m = \frac{f(9) - f(4)}{9-4} = \frac{9-7}{5} = \frac{2}{5}$$

(B) By the Mean Value Theorem, we know there exists at least one c in the open interval $(4, 9)$ such that $f'(c)$ is equal to this mean slope. Find all values of c that work.

$$f'(x) = 2\left(\tfrac{1}{2}\right)x^{-1/2} = \frac{1}{\sqrt{x}}$$

Want: $f'(c) = m$

$$\frac{1}{\sqrt{c}} = \frac{2}{5}$$
$$\sqrt{c} = \frac{5}{2}$$
$$c = \frac{25}{4}$$

---

20. Evaluate the limit.

$$\lim_{x\to\infty}\left(\frac{10x}{10x+3}\right)^{5x}$$

Let $y = \lim_{x\to\infty}\left(\frac{10x}{10x+3}\right)^{5x}$. "$1^\infty$"

$\ln y = \lim_{x\to\infty} 5x \ln\left(\frac{10x}{10x+3}\right)$ "$\infty \cdot 0$"

$= 5\lim_{x\to\infty} \frac{\ln\left(\frac{10x}{10x+3}\right)}{\left(\frac{1}{x}\right)}$ "$\frac{0}{0}$"

$\overset{L'H}{=} 5\lim_{x\to\infty} \frac{\frac{10x+3}{10x}\cdot\frac{(10x+3)(10)-(10x)(10)}{(10x+3)^2}}{-\frac{1}{x^2}}$

$= -5\lim_{x\to\infty} \frac{\left(\frac{3}{x(10x+3)}\right)}{\left(\frac{1}{x^2}\right)}$

$\ln y = -5\lim_{x\to\infty} \frac{3x^2}{x(10x+3)}$

$= -15\lim_{x\to\infty} \frac{x}{10x+3}$

$= -15\lim_{x\to\infty} \frac{1}{10+\frac{3}{x}}$

$= -15 \cdot \frac{1}{10}$

$\ln y = -\frac{3}{2}$

$y = e^{-3/2}$

21. Evaluate the following limits.

(A) $\lim_{x\to 1} \frac{6^x - 6}{x^2 - 1}$ "$\frac{0}{0}$"

$\overset{L'H}{=} \lim_{x\to 1} \frac{(\ln 6) 6^x}{2x}$

$= \frac{(\ln 6) 6^1}{2(1)}$

$= 3 \ln 6$

(B) $\lim_{x\to\infty} \frac{\tan^{-1}(x)}{(1/x) - 6}$ (not indeterminate)

Note:

$\lim_{x\to\infty} \tan^{-1}(x) = \frac{\pi}{2}$

and

$\lim_{x\to\infty} \left(\frac{1}{x} - 6\right) = -6$

So, $\lim_{x\to\infty} \frac{\tan^{-1}(x)}{\frac{1}{x} - 6} = \frac{(\pi/2)}{-6} = -\frac{\pi}{12}$.

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22. Evaluate the following limit using L'Hospital's rule if appropriate.

$\lim_{x\to\infty} (\sqrt[3]{x^3 - 7x^2} - x)$ "$\infty - \infty$"

$= \lim_{x\to\infty} \left(\sqrt[3]{x^3(1 - \frac{7}{x})} - x\right)$

$= \lim_{x\to\infty} \left(x \sqrt[3]{1 - \frac{7}{x}} - x\right)$

$= \lim_{x\to\infty} x \left(\sqrt[3]{1 - \frac{7}{x}} - 1\right)$ "$\infty \cdot 0$"

$= \lim_{x\to\infty} \frac{\sqrt[3]{1 - \frac{7}{x}} - 1}{\left(\frac{1}{x}\right)}$ "$\frac{0}{0}$"

$\overset{L'H}{=} \lim_{x\to\infty} \frac{\frac{1}{3}\left(1 - \frac{7}{x}\right)^{-2/3} (-7)(-1)x^{-2}}{(-1)x^{-2}}$

$= \lim_{x\to\infty} \frac{-7}{3\left(1 - \frac{7}{x}\right)^{2/3}}$

$= \frac{-7}{3}$

23. Suppose f is twice differentiable with

$$f''(x) = 7x - 2, \quad f'(-2) = 0, \quad \text{and} \quad f(-2) = -2.$$

Find $f'(x)$ and find $f(3)$.

$$f'(x) = \int f''(x)\,dx = \int (7x-2)\,dx = \frac{7}{2}x^2 - 2x + C_1$$

$$0 = f'(-2) = \frac{7}{2}(-2)^2 - 2(-2) + C_1 = 14 + 4 + C_1 = 18 + C_1$$

$$0 = 18 + C_1$$

$$C_1 = -18$$

$$f'(x) = \frac{7}{2}x^2 - 2x - 18$$

$$f(x) = \int f'(x)\,dx = \int \left(\frac{7}{2}x^2 - 2x - 18\right)dx = \frac{7}{6}x^3 - x^2 - 18x + C_2$$

$$-2 = f(-2) = \frac{7}{6}(-2)^3 - (-2)^2 - 18(-2) + C_2 = -\frac{28}{3} - 4 + 36 + C_2 = \frac{68}{3} + C_2$$

$$-2 = \frac{68}{3} + C_2$$

$$C_2 = -\frac{74}{3}$$

$$f(x) = \frac{7}{6}x^3 - x^2 - 18x - \frac{74}{3} \qquad f(3) = \frac{7}{6}(3)^3 - (3)^2 - 18(3) - \frac{74}{3} = -\frac{337}{6}$$

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24. A car traveling at 49 ft/sec decelerates at a constant 4 feet per second squared. How many feet does the car travel before coming to a complete stop?

$$a(t) = -4 \qquad v(0) = 49 \qquad s(0) = 0$$

$a(t)$ = acceleration at time t

$v(t)$ = velocity at time t

$s(t)$ = position at time t

t_0 = time when velocity is 0

$$v(t) = \int a(t)\,dt = \int -4\,dt = -4t + C_1$$

$$49 = v(0) = -4(0) + C_1 \Rightarrow C_1 = 49$$

So, $v(t) = -4t + 49$.

We want $s(t_0)$, where t_0 is the time when $v(t_0) = 0$.

$$s(t) = \int v(t)\,dt = \int (-4t + 49)\,dt = -2t^2 + 49t + C_2$$

$$0 = s(0) = -2(0)^2 + 49(0) + C_2 = C_2 \Rightarrow 0 = C_2$$

So, $s(t) = -2t^2 + 49t$.

Find t_0: $0 = v(t_0) = -4t_0 + 49 \Rightarrow t_0 = \frac{49}{4}$

$$s(t_0) = -2\left(\frac{49}{4}\right)^2 + 49\left(\frac{49}{4}\right) = 300.125 \text{ ft.} \quad \left(= \frac{2401}{8}\right)$$