1. Let $f(x) = x^2 + 1$. Use a right Riemann sum with $n = 6$ to estimate $\int_{-3}^{3} f(x)\, dx$.

For a Right Riemann Sum: $x_k^* = a + k\Delta x = -3 + k$

$\Delta x = \frac{b-a}{n} = \frac{3-(-3)}{6} = 1$

$$\sum_{k=1}^{n} f(x_k^*)\Delta x = f(-2)\,\Delta x + f(-1)\Delta x + f(0)\Delta x + f(1)\Delta x + f(2)\,\Delta x + f(3)\Delta x$$

$$= (5)(1) + (2)(1) + (1)(1) + (2)(1) + (5)(1) + (10)(1)$$

$$= 25$$

---

2. Compute the derivative below. $\frac{d}{dx}\left[\int_{x^3}^{2} \frac{t}{\sqrt{1+t^2}}\, dt\right]$

Let $A(x) = \int_2^x \frac{t}{\sqrt{1+t^2}}\, dt$.

then $A'(x) = \frac{x}{\sqrt{1+x^2}}$.

$$= \frac{d}{dx}\left[-\int_2^{x^3} \frac{t}{\sqrt{1+t^2}}\, dt\right]$$

$$= \frac{d}{dx}\left[-A(x^3)\right] = -A'(x^3)\,\frac{d}{dx}(x^3)$$

$$= -\frac{x^3}{\sqrt{1+(x^3)^2}} \cdot 3x^2$$

3. Let $f(x)$ and $g(x)$ be integrable functions with the properties given below.

$$\int_1^2 f(x)\,dx = -5 \qquad \int_2^5 g(x)\,dx = 1$$

$$\int_1^7 f(x)\,dx = 4 \qquad \int_5^7 g(x)\,dx = -2$$

Compute $\int_2^7 (2f(x) - g(x) + 1)\,dx$.

$$= 2\int_2^7 f(x)\,dx - \int_2^7 g(x)\,dx + \int_2^7 dx$$

$$= 2\left[\int_1^7 f(x)dx - \int_1^2 f(x)dx\right] - \left[\int_2^5 g(x)dx + \int_5^7 g(x)dx\right] + [x]_2^7$$

$$= 2(4-(-5)) - (1+(-2)) + (7-2)$$

$$= 18 - (-1) + 5$$

$$= 24$$

---

4. Evaluate the definite integral. $\int_{-2}^{3} (x^2 - x - 6)\,dx$

$$= \left[\tfrac{1}{3}x^3 - \tfrac{1}{2}x^2 - 6x\right]_{-2}^{3}$$

$$= \left(\tfrac{27}{3} - \tfrac{9}{2} - 18\right) - \left(-\tfrac{8}{3} - \tfrac{4}{2} + 12\right)$$

$$= -\frac{125}{6}$$

5. Evaluate the definite integral. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\cos x - 1)\,dx$

<u>Note</u>: the interval is symmetric about 0.

The integrand is <u>even</u>, so

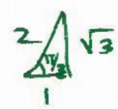
$$= 2\int_0^{\frac{\pi}{2}} (\cos x - 1)\,dx$$

$$= 2\left[\sin x - x\right]_0^{\pi/2}$$

$$= 2\left[\left(\sin\tfrac{\pi}{2} - \tfrac{\pi}{2}\right) - (\sin 0 - 0)\right]$$

$$= 2\left(1 - \tfrac{\pi}{2}\right) = 2 - \pi$$

6. Evaluate the definite integral. $\int_1^{\sqrt{3}} \frac{dx}{1+x^2}$



$$= \left[\tan^{-1} x\right]_1^{\sqrt{3}}$$

$$= \tan^{-1}(\sqrt{3}) - \tan^{-1}(1)$$

$$= \frac{\pi}{3} - \frac{\pi}{4}$$

$$= \frac{\pi}{12}$$

7. The height of an arch above the ground is given by the function $y = 12\sin(\frac{\pi x}{4})$ for $0 \le x \le 4$ What is the average height of the arch above the ground?

($12\sin(\frac{\pi x}{4})$ is $f(x)$)

avg. value of f on $[a,b]$ $= \bar{f} = \frac{1}{b-a}\int_a^b f(x)\,dx$

$$= \frac{1}{4-0}\int_0^4 12\sin\left(\frac{\pi x}{4}\right)dx$$

$$= \frac{1}{4}\cdot 12\int_0^4 \sin\left(\frac{\pi x}{4}\right)dx$$

$$= 3\left[-\frac{4}{\pi}\cos\left(\frac{\pi x}{4}\right)\right]_0^4$$

$$= 3\left[-\frac{4}{\pi}\cos(\pi) - \left(-\frac{4}{\pi}\cos(0)\right)\right] = \frac{24}{\pi}$$

8. Compute the following indefinite integral. $\int \frac{2x^2}{\sqrt{1-4x^3}}\,dx$

$u = 1-4x^3$
$du = -12x^2\,dx$
$-\frac{1}{6}du = 2x^2dx$

$$= \int \frac{1}{\sqrt{u}}\left(-\frac{1}{6}du\right)$$

$$= -\frac{1}{6}\int \frac{1}{\sqrt{u}}\,du$$

$$= -\frac{1}{6}\int u^{-1/2}\,du$$

$$= -\frac{1}{6}\left[2u^{1/2}\right] + C$$

$$= -\frac{1}{3}\sqrt{1-4x^3} + C$$

---

9. Compute the following indefinite integral. $\int \frac{x}{\sqrt{x-4}}\,dx$

$u = x-4 \qquad x = u+4$
$du = dx$

$$= \int \frac{u+4}{\sqrt{u}}\,du$$

$$= \int \left(\frac{u}{\sqrt{u}} + \frac{4}{\sqrt{u}}\right) du$$

$$= \int \left(u^{1/2} + 4u^{-1/2}\right] du$$

$$= \left[\frac{2}{3}u^{3/2} + 8u^{1/2}\right] + C$$

$$= \frac{2}{3}(x-4)^{3/2} + 8(x-4)^{1/2} + C$$

10. Compute the following definite integral. $\int_0^{\frac{\pi}{4}} \frac{\sin x}{\cos^2 x}\, dx$

$$u = \cos x$$
$$du = -\sin x \, dx$$
$$-du = \sin x \, dx$$
$$@\ x = 0,\quad u = 1$$
$$@\ x = \frac{\pi}{4},\quad u = \frac{\sqrt{2}}{2}$$

$$= \int_1^{\frac{\sqrt{2}}{2}} \frac{1}{u^2}(-du)$$
$$= -\int_1^{\sqrt{2}/2} \frac{1}{u^2}\, du$$
$$= -\int_1^{\sqrt{2}/2} u^{-2}\, du$$
$$= -\left[-u^{-1}\right]_1^{\sqrt{2}/2}$$
$$= \frac{1}{\left(\frac{\sqrt{2}}{2}\right)} - \frac{1}{1} = \sqrt{2} - 1$$

---

11. Compute the following definite integral. $\int_0^{\ln 4} \frac{e^x}{3 + 2e^x}\, dx$

$$u = 3 + 2e^x$$
$$du = 2e^x\, dx$$
$$\frac{1}{2}du = e^x dx$$
$$@\ x = 0,\ u = 5$$
$$@\ x = \ln 4,\quad u = 11$$

$$= \int_5^{11} \frac{1}{u}\left(\frac{1}{2}du\right)$$
$$= \frac{1}{2}\int_5^{11} \frac{1}{u}\, du$$
$$= \frac{1}{2}\left[\ln|u|\right]_5^{11}$$
$$= \frac{1}{2}\left[\ln 11 - \ln 5\right]$$
$$= \frac{1}{2}\ln\left(\frac{11}{5}\right)$$

12. Find the position s and velocity v of an object moving along a straight line with the given acceleration, initial velocity, and initial position.

$a(t) = e^{-t} \qquad v(0) = 6 \qquad s(0) = 4$

$$v(t) - v(0) = \int_0^t a(x)\,dx = \int_0^t e^{-x}\,dx = \left[-e^{-x}\right]_0^t$$

$$v(t) = v(0) + \left[(-e^{-t}) - (-e^{0})\right]$$

$$= 6 - e^{-t} + 1$$

$$= 7 - e^{-t}$$

$$s(t) - s(0) = \int_0^t v(x)\,dx = \int_0^t (7 - e^{-x})\,dx = \left[7x + e^{-x}\right]_0^t$$

$$s(t) = s(0) + \left[(7t + e^{-t}) - (7\cdot 0 + e^{0})\right]$$

$$= 4 + 7t + e^{-t} - 1$$

$$= 3 + 7t + e^{-t}$$

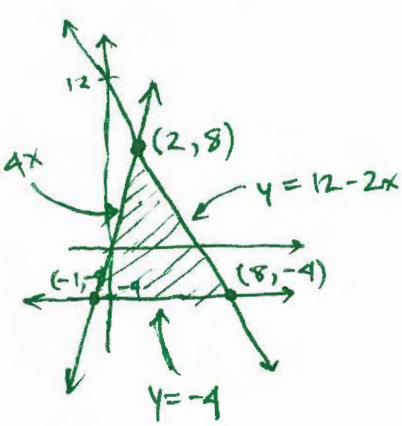
---

13. A region is bounded by the curves $y = 4x$, $y = -4$, and $y = 12 - 2x$.

(a) Use vertical slices to express the area of this region using an integral or sum of integrals.

(b) Use horizontal slices to express the area of this region using an integral or sum of integrals.

(c) Find the area of this region.



(a) $A = \int_{-1}^{2} \left[(4x) - (-4)\right]dx + \int_{2}^{8} \left[(12-2x) - (-4)\right]dx$

General form for vertical slices: $\int_a^b \left[(\text{upper}) - (\text{lower})\right]dx$

(b) $A = \int_{-4}^{8} \left[\left(\frac{12-y}{2}\right) - \left(\frac{1}{4}y\right)\right]dy$

General form for horiz. slices: $\int_c^d \left[(\text{right}) - (\text{left})\right]dy$

$y = 12 - 2x \Leftrightarrow y - 12 = -2x$

$\frac{12-y}{2} = x$

$y = 4x \Rightarrow x = \frac{1}{4}y$

(c) $A = \int_{-4}^{8} \left[6 - \frac{1}{2}y - \frac{1}{4}y\right]dy$

$= \int_{-4}^{8} \left[6 - \frac{3}{4}y\right]dy$

$= \left[6y - \frac{3}{8}y^2\right]_{-4}^{8}$

$= \left[\left(6\cdot 8 - \frac{3}{8}\cdot 8^2\right) - \left(6(-4) - \frac{3}{8}(-4^2)\right)\right]$

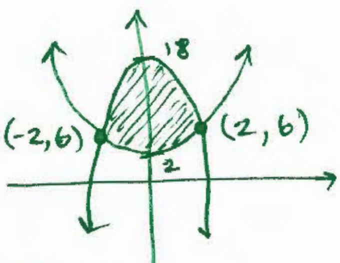
A = 54

14. A region is bounded by the curves $y = 18 - 3x^2$ and $y = x^2 + 2$.

(a) Use vertical slices to express the area of this region using an integral or sum of integrals.

(b) Use horizontal slices to express the area of this region using an integral or sum of integrals.

(c) Find the area of this region.



$18 - 3x^2 = x^2 + 2$

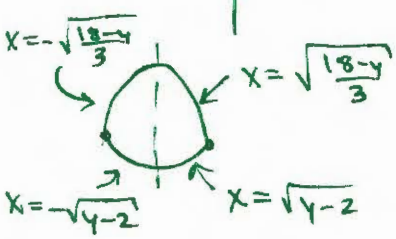
$16 = 4x^2$

$4 = x^2$

$x = -2, 2$

(a) $A = \int_{-2}^{2} \left[(18 - 3x^2) - (x^2 + 2) \right] dx$

(b) $A = \int_{2}^{6} \left[(\sqrt{y-2}) - (-\sqrt{y-2}) \right] dy + \int_{6}^{18} \left[\left(\sqrt{\frac{18-y}{3}}\right) - \left(-\sqrt{\frac{18-y}{3}}\right) \right] dy$



(c) $A = \int_{-2}^{2} \left[16 - 4x^2 \right] dx$

$= \left[16x - \frac{4}{3}x^3 \right]_{-2}^{2}$

$= \left[(16 \cdot 2 - \frac{4}{3} \cdot 2^3) - (16(-2) - \frac{4}{3}(-2)^3) \right]$

$= \frac{128}{3}$

$y = 18 - 3x^2$

$y - 18 = -3x^2$

$x^2 = \frac{18-y}{3}$

$x = \pm\sqrt{(18-y)/3}$

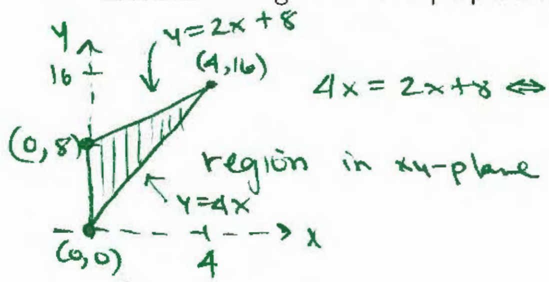
$y = x^2 + 2$

$x^2 = y - 2$

$x = \pm\sqrt{y-2}$

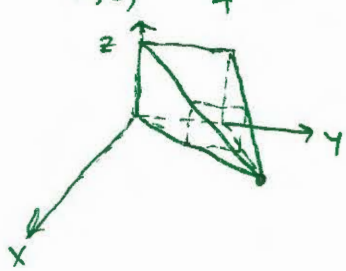
---

15. The base of a solid is the region in the xy-plane bounded by $y = 4x$, $y = 2x + 8$, and $x = 0$. The cross sections through the solid perpendicular to the x-axis are squares. Find the volume of the solid.



$4x = 2x + 8 \Leftrightarrow 2x = 8$

$x = 4$



$V = \int_a^b A(x)\, dx$ ← cross sectional area function

$= \int_0^4 \left[(2x+8) - (4x) \right]^2 dx$

$= \int_0^4 \left[8 - 2x \right]^2 dx$

$= \int_0^4 (64 - 32x + 4x^2)\, dx$

$= \left[64x - 16x^2 + \frac{4}{3}x^3 \right]_0^4$

$= 64 \cdot 4 - 16 \cdot 4^2 + \frac{4}{3} \cdot 4^3$

$= \frac{256}{3}$

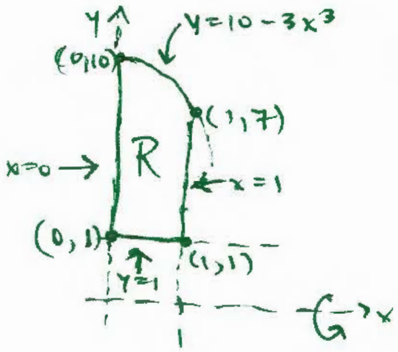
16. The region $\mathcal{R}$ is bounded by the curves $y = 10 - 3x^3$, $y = 1$, $x = 0$ and $x = 1$.

(a) A solid is formed by revolving $\mathcal{R}$ about the x-axis.

i. Use the Disk/Washer Method to express the volume of this solid as an integral or sum of integrals.

ii. Use the Shell Method to express the volume of this solid as an integral or sum of integrals.

iii. Find the volume of this solid.



(i) $V = \int_0^1 \left[\pi(10-3x^3)^2 - \pi(1)^2\right] dx$

(ii) $\int_1^7 2\pi y\left[(1)-(0)\right]dy + \int_7^{10} 2\pi y\left[\left(\sqrt[3]{\frac{10-y}{3}}\right)-(0)\right] dy$

$y = 10 - 3x^3$

$3x^3 = 10 - y$

$x = \sqrt[3]{\frac{10-y}{3}}$

(iii) $V = \pi\int_0^1 \left[100 - 60x^3 + 9x^6 - 1\right] dx$

$= \pi\left[99x - \frac{60}{4}x^4 + \frac{9}{7}x^7\right]_0^1$

$= \pi\left(99 - 15 + \frac{9}{7}\right)$

$= \frac{597\pi}{7}$

(b) A solid is formed by revolving $\mathcal{R}$ about the y-axis.

i. Use the Disk/Washer Method to express the volume of this solid as an integral or sum of integrals.

ii. Use the Shell Method to express the volume of this solid as an integral or sum of integrals.

iii. Find the volume of this solid.

(i) $V = \int_1^7 \pi(1)^2\, dy + \int_7^{10} \pi\left(\sqrt[3]{\frac{10-y}{3}}\right)^2 dy$

(ii) $V = \int_0^1 2\pi x\left[(10-3x^3)-(1)\right] dx$

(iii) $V = 2\pi\int_0^1 x(9-3x^3)\,dx$

$= 2\pi\int_0^1 (9x - 3x^4)\,dx$

$= 2\pi\left[\frac{9}{2}x^2 - \frac{3}{5}x^5\right]_0^1$

$= 2\pi\left(\frac{9}{2} - \frac{3}{5}\right)$

$= \frac{39\pi}{5}$

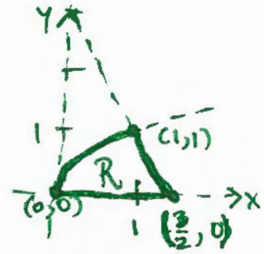
17. The region $\mathcal{R}$ is bounded by the curves $y = \sqrt{x}$, $y = 3 - 2x$, and $y = 0$.

(a) A solid is formed by revolving $\mathcal{R}$ about the x-axis.

i. Use the Disk/Washer Method to express the volume of this solid as an integral or sum of integrals.

ii. Use the Shell Method to express the volume of this solid as an integral or sum of integrals.

iii. Find the volume of this solid.



intersections:

$y=\sqrt{x}$ $\quad y = 3-2x$

$y^2 = x$ $\quad y = 3 - 2y^2$

$y \geq 0$ $\quad 2y^2 + y - 3 = 0$

$(y-1)(2y+3) = 0$

$y = 1$ or ~~$y = -\frac{3}{2}$~~

b/c $y \geq 0$

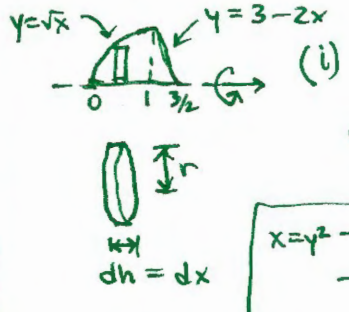
$y = 0$ $\quad y = 3-2x$

$0 = 3 - 2x$

$2x = 3$

$x = \frac{3}{2}$

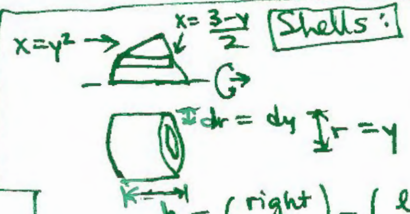
Disks:



(i) $V = \int_a^b \pi r^2 \, dh + \int_b^c \pi r^2 \, dh$

$V = \int_0^1 \pi(\sqrt{x})^2 dx + \int_1^{3/2} \pi (3-2x)^2 dx$

Shells:



(ii) $V = \int 2\pi r h \, dr$

$V = \int_0^1 2\pi y \left(\frac{3-y}{2} - y^2\right) dy$

$h = (\text{right } x) - (\text{left } x)$

$= \frac{3-y}{2} - y^2$

(iii) $V = \int_0^1 2\pi y \left(\frac{3}{2} - \frac{1}{2}y - y^2\right) dy = 2\pi \int_0^1 \left(\frac{3}{2}y - \frac{1}{2}y^2 - y^3\right) dy$

$= 2\pi \left[\frac{3}{4}y^2 - \frac{1}{6}y^3 - \frac{1}{4}y^4\right]_0^1$

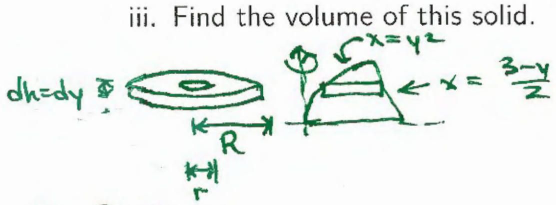
$= 2\pi \left(\frac{3}{4} - \frac{1}{6} - \frac{1}{4}\right) = \frac{2\pi}{3}$

(b) A solid is formed by revolving $\mathcal{R}$ about the y-axis.

i. Use the Disk/Washer Method to express the volume of this solid as an integral or sum of integrals.

ii. Use the Shell Method to express the volume of this solid as an integral or sum of integrals.

iii. Find the volume of this solid.



$R = \frac{3-y}{2}$

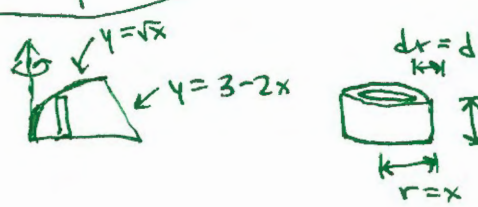
$r = y^2$

Washers:

(i) $V = \int [\pi R^2 - \pi r^2] \, dh$

$= \int_0^1 \left[\pi\left(\frac{3-y}{2}\right)^2 - \pi (y^2)^2\right] dy$

Shells:



(ii) $V = \int 2\pi r h \, dr + \int 2\pi r h \, dr$

$V = \int_0^1 2\pi x (\sqrt{x}) \, dx + \int_1^{3/2} 2\pi x (3-2x) \, dx$

(iii) $V = \int_0^1 \pi \left[\frac{1}{4}(9 - 6y + y^2) - y^4\right] dy = \pi \int_0^1 \left(\frac{9}{4} - \frac{3}{2}y + \frac{1}{4}y^2 - y^4\right) dy$

$= \pi \left[\frac{9}{4}y - \frac{3}{4}y^2 + \frac{1}{12}y^3 - \frac{1}{5}y^5\right]_0^1$

$= \pi \left(\frac{9}{4} - \frac{3}{4} + \frac{1}{12} - \frac{1}{5}\right) = \frac{83\pi}{60}$

18. Find the length of the segment of the curve $y = 1 + 5x^{\frac{3}{2}}$ from $x = 0$ to $x = 4$.

$\frac{dy}{dx} = \frac{15}{2}x^{1/2}$

$ds = \sqrt{1 + (\frac{dy}{dx})^2}\, dx$

$= \sqrt{1 + \frac{225}{4}x}\, dx$

$L = \int_a^b ds$

$= \int_0^4 \sqrt{1 + \frac{225}{4}x}\, dx$

$= \int_1^{226} \sqrt{u} \cdot \frac{4}{225}\, du$

$= \frac{4}{225}\left[\frac{2}{3}u^{3/2}\right]_1^{226}$

$= \frac{4}{225}\left[\frac{2}{3}(226)^{3/2} - \frac{2}{3}\right]$

$u = 1 + \frac{225}{4}x$

$du = \frac{225}{4}dx$

$\frac{4}{225}du = dx$

@ $x = 0$, $u = 1$

@ $x = 4$, $u = 226$

---

19. Find the length of the segment of the curve $x = \frac{3}{4}y^4 + \frac{1}{24}y^{-2}$ from $y = 1$ to $y = 2$.

$\frac{dx}{dy} = 3y^3 - \frac{1}{12}y^{-3}$

$ds = \sqrt{1 + (\frac{dx}{dy})^2}\, dy$

$= \sqrt{1 + (3y^3 - \frac{1}{12}y^{-3})^2}\, dy$

$= \sqrt{1 + 9y^6 + 2(3y^3)(-\frac{1}{12}y^{-3}) + \frac{1}{144}y^{-6}}\, dy$

$= \sqrt{1 + 9y^6 - \frac{1}{2} + \frac{1}{144}y^{-6}}\, dy$

$= \sqrt{9y^6 + \frac{1}{2} + \frac{1}{144}y^{-6}}\, dy$

$= \sqrt{(3y^3 + \frac{1}{12}y^{-3})^2}\, dy$

$= (3y^3 + \frac{1}{12}y^{-3})\, dy$

(b/c $y \geq 0$)

$L = \int_c^d ds$

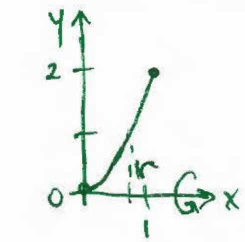
$= \int_1^2 (3y^3 + \frac{1}{12}y^{-3})\, dy$

$= \left[\frac{3}{4}y^4 - \frac{1}{24}y^{-2}\right]_1^2$

$= (12 - \frac{1}{92}) - (\frac{3}{4} - \frac{1}{24})$

$= \frac{361}{32}$

20. A surface of revolution is formed by revolving the curve $y = 2x^3$ from $x = 0$ to $x = 1$ about the x-axis. Find the surface area.



$r = f(x) = 2x^3$

$f'(x) = 6x^2$

$$SA = \int_a^b 2\pi r\, ds$$
$$= \int_0^1 2\pi f(x)\sqrt{1+(f'(x))^2}\, dx$$
$$= 2\pi \int_0^1 2x^3 \sqrt{1+(6x^2)^2}\, dx$$
$$= 4\pi \int_0^1 x^3 \sqrt{1+36x^4}\, dx$$

$u = 1+36x^4$

$du = 144x^3\, dx$

$\frac{1}{144} du = x^3\, dx$

@ $x=0$, $u=1$

@ $x=1$, $u=37$

$$= 4\pi \int_1^{37} \sqrt{u} \cdot \frac{1}{144}\, du$$
$$= \frac{\pi}{36} \int_1^{37} \sqrt{u}\, du$$
$$= \frac{\pi}{36} \left[\frac{2}{3} u^{3/2} \right]_1^{37}$$
$$= \frac{\pi}{36} \left[\frac{2}{3}(37)^{3/2} - \frac{2}{3} \right]$$

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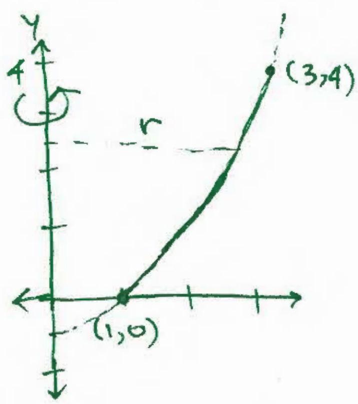
21. A surface of revolution is formed by revolving the curve $x = \sqrt{2y+1}$ from $x = 1$ to $x = 3$ about the y-axis. Find the surface area.



$r = g(y) = \sqrt{2y+1}$

$g'(y) = \frac{1}{2\sqrt{2y+1}} \cdot 2$

$= \frac{1}{\sqrt{2y+1}}$

$$SA = \int_c^d 2\pi r\, ds$$
$$= \int_0^4 2\pi g(y)\sqrt{1+(g'(y))^2}\, dy$$
$$= 2\pi \int_0^4 \sqrt{2y+1}\sqrt{1+\left(\frac{1}{\sqrt{2y+1}}\right)^2}\, dy$$
$$= 2\pi \int_0^4 \sqrt{2y+1}\sqrt{1+\frac{1}{2y+1}}\, dy$$
$$= 2\pi \int_0^4 \sqrt{(2y+1)\left[1+\frac{1}{2y+1}\right]}\, dy$$
$$= 2\pi \int_0^4 \sqrt{2y+1+1}\, dy$$
$$= 2\pi \int_0^4 \sqrt{2y+2}\, dy$$

$u = 2y+2$

$du = 2dy$

$\frac{1}{2} du = dy$

@ $y=0$, $u=2$

@ $y=4$, $u=10$

$$= 2\pi \int_2^{10} \sqrt{u} \cdot \frac{1}{2}\, du$$
$$= \pi \left[\frac{2}{3} u^{3/2} \right]_2^{10}$$
$$= \pi \left[\frac{2}{3}(10)^{3/2} - \frac{2}{3}(2)^{3/2} \right]$$

22. A thin rod from $x = 1$ to $x = b$ (in m) has density $\rho(x) = x$ (in kg/m).

(a) What is the mass of the rod in terms of b.

(b) If the rod has a mass of 40 kg, what is b? How long is the rod?

Note: density cannot be negative, so x-values are ≥ 0. That is, $b \geq 0$.

(a) $m = \int_1^b \rho(x)\,dx$

$= \int_1^b x\,dx$

$= \left[\frac{1}{2}x^2\right]_1^b$

$m = \frac{1}{2}b^2 - \frac{1}{2}$

(b) $40 = m = \frac{1}{2}b^2 - \frac{1}{2}$

$80 = b^2 - 1$

$81 = b^2$

$b = 9$ or ~~$b = -9$~~

length $= b - a = 9 - 1 = 8$ m.

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23. A spring with spring constant k requires 80 N to hold in a stretched position 2 m past its equilibrium position.

(a) How much work was required to stretch the spring?

(b) How much additional work is required to stretch the spring an additional 2 m?

$80 = k \cdot 2$

$k = 40$

(a) $W = \int_0^2 F\,dx = \int_0^2 40x\,dx = \left[20x^2\right]_0^2$

$W = 80$ J

(b) $W = \int_2^4 F\,dx = \int_2^4 40x\,dx = \left[20x^2\right]_2^4$

$W = 320 - 80 = 240$ J

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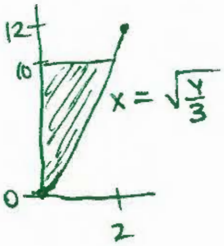
24. A tank is formed by revolving the parabolic segment $y = 3x^2$ from $x = 0$ to $x = 2$ (in m) about the y-axis. The tank is filled to a depth of 10 m with water ($\rho = 1000\,\text{kg/m}^3$). How much work is required to pump all the water to the level of the top of the tank and out of the tank?



$W = \int_a^b \rho g\, A(y)\, D(y)\,dy$

$= \int_0^{10} (1000)(9.8)\left(\frac{\pi}{3}y\right)(12 - y)\,dy$

$= \frac{980\pi}{3} \int_0^{10} (12y - y^2)\,dy$

$= \frac{980\pi}{3} \left[6y^2 - \frac{1}{3}y^3\right]_0^{10}$

$= \frac{980\pi}{3}\left(600 - \frac{1000}{3}\right)$ J

$D(y) = 12 - y$

$A(y) = \pi x^2 = \pi\left(\sqrt{\frac{y}{3}}\right)^2$

$A(y) = \frac{\pi}{3}y$