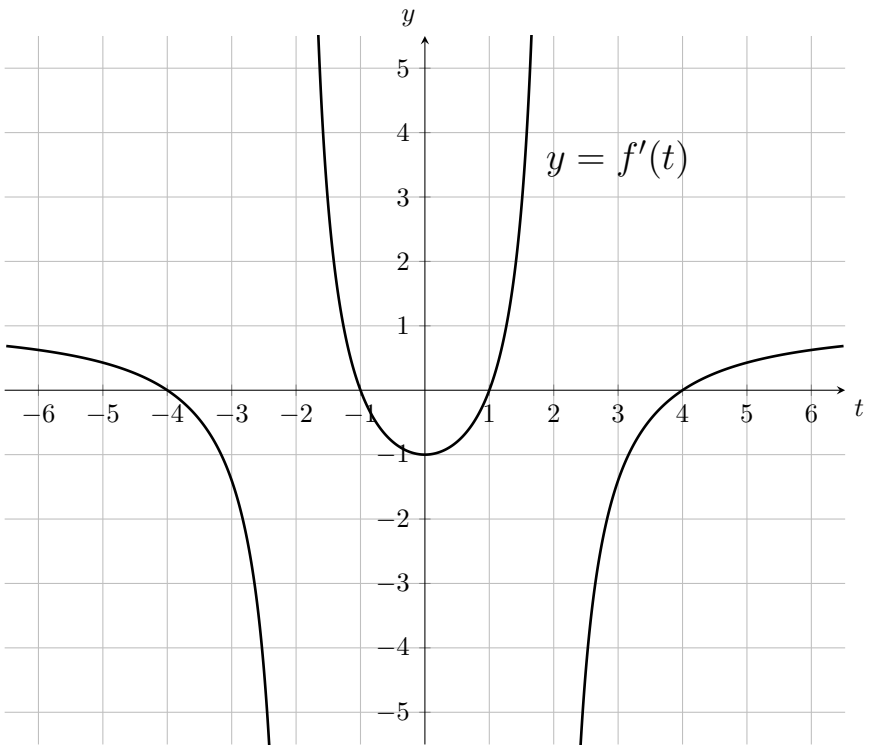
Practice Midterm 2 – Math 1161.0X

1. Decide if the following statements are TRUE or FALSE. **You do NOT need to justify your answers.**
   (a) (1 point) If $f'(c) = 0$ then f must have a local minimum or a local maximum at c.
   (b) (1 point) If f is continuous on the closed interval $[0, 7]$ then f has a global maximum and a global mininum on $[0, 7]$.
   (c) (1 point) $f(x) = x^3$ has both a critical point and an inflection point at $x = 0$.
   (d) (1 point) If f is nondecreasing on the interval $[2, 5]$ and nondecreasing on the interval $[5, 10]$ then it is nondecreasing on the interval $[2, 10]$.
   (e) (1 point) If f is an even function then f has a critical point at $x = 0$.

2. Give examples of the following. Be as explicit as possible. **You do NOT need to justify your answers.**
   (a) (2 points) Give an example of a function $f(x)$ with domain $(-\infty, \infty)$ which is both odd and even.
   (b) (2 points) Give an example of a continuous function $f(x)$ with domain $(-\infty, \infty)$ which has no local extrema.
   (c) (2 points) Give an example of a function $f(x)$ which is decreasing and concave up on the interval $(-\infty, \infty)$.

3. The graph the **derivative** $y = f'(t)$ is given below.



   (a) (3 points) At what t-values does the function f whose **derivative** is pictured above have critical points?
   (b) (3 points) What are the intervals of increase for the function f?
   (c) (3 points) What are the intervals of decrease for f?
   (d) (3 points) At what t-values does f have an inflection point.
   (e) (3 points) On what open intervals is f concave up?

(f) (3 points) On what open intervals is f concave down?

4. Consider the curve C in the xy-plane satisfying the equation

$$x^3y - x^2y^2 + xy^3 = -10$$

(a) (3 points) Compute $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y.

(b) (3 points) What is the equation of the tangent line to the curve C at the point $(2, -1)$?

5. (5 points) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ if $y = \dfrac{(x-5)^2 \tan^4 x}{(\cos x + 2)(x^4 + 10)}$. You **do not need to simplify** your answer.

6. Let

$$f(x) = \arctan(x^2)$$

(a) (3 points) What is the domain of f?

(b) (3 points) Is f even, odd, periodic?

(c) (3 points) Compute $f'(x)$.

(d) (5 points) What are x-coordinates for the critical points of f?

(e) (5 points) What are the intervals of increase for f?

(f) (5 points) What are the intervals of decrease for f?

(g) (5 points) What is the second derivative of f?

(h) (5 points) On what open intervals is f concave up?

(i) (5 points) On what open intervals is f concave down?

(j) (5 points) Does f have any horizontal or vertical asymptoptes? If so what are their equations?

(k) (5 points) Graph f

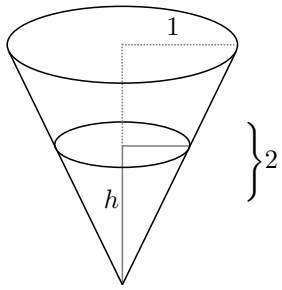
7. Evaluate the following limits using any technique you like.

(a) (5 points) $\lim\limits_{x \to 0} \dfrac{3x^2}{\sin x}$

(b) (5 points) $\lim\limits_{x \to \infty} \sqrt[x]{x}$

(c) (5 points) $\lim\limits_{x \to \infty} \left(1 + \dfrac{2}{x}\right)^x$

8. Water is pouring into a cone with a height of 2 meters and a radius of 1 meter at a rate of $3\ \mathrm{m}^3/\mathrm{s}$.



(a) (5 points) Give an equation relating the height h of the water to the volume V of water in the cone. *(Hint: The volume of a cone with height h and radius r is $\frac{1}{3}\pi r^2 h$.)*

(b) (5 points) When the height of the water is 1 meter how fast is the water level rising.

9. The height h and radius r of a circular cylinder are both greater than or equal to 0 and have a sum of 1 meter.

   (a) (5 points) Express the volume V of the cylinder as a function of its radius r. Include the **domain** for the radius. *(Hint: The volume of a cylinder with height h and radius r is $\pi r^2 h$.)*

   (b) (5 points) Is there a radius in the domain above which gives the maximum volume for the cylinder? Why or why not? In particular do any theorems from class or the book apply to this situation?

   (c) (5 points) Find the radius r of the cylinder with the maximum volume.

   (d) (5 points) What is the maximum volume?