Practice Midterm 3 – Math 1161.0X

1. Decide if the following statements are TRUE or FALSE. **You do NOT need to justify your answers.**
   (a) (1 point) If f is an odd function then $\int_{-100}^{100} |f(x)|dx = 0$
   (b) (1 point) The arc length of $y = \sqrt{4 - x^2}$ from $-2 \leq x \leq 2$ is 2π
   (c) (1 point) If $\int_0^9 f(x)dx = 12$ then $\int_0^3 xf(x^2)dx = 6$
   (d) (1 point) If f is a function defined over all real numbers with $f'(x) > 0$, then a left end point Riemann sum of f would be an overestimate of $\int_0^{100} f(x)dx$
   (e) (1 point) If f is a continuous function on $[a, b]$ and $\int_a^b f(x) = 0$ then $f(c) = 0$ for some c in $[a, b]$.

2. Answer the following short answer type questions:
   (a) (2 points) A car starts moving straight ahead from point A at $t = 0$ with velocity $v(t) = t^2 + 1$. What is its displacement from A when $t = 3$?
   (b) (2 points) Compute $\frac{d}{dx}\int_e^{x^3+4x+2} \ln(t - \sqrt{t^2 - 1})dx$
   (c) (2 points) Give an example of an odd function $f(x)$ defined over all real numbers such that $\int_0^1 f(x) = 10$.

3. Evaluate the following integrals:
   (a) $\displaystyle\int \frac{3\sec^2(x)}{(1 + \tan(x))^3}dx$
   (b) $\displaystyle\int_{-3}^{3} x\sqrt{9 - x^2}dx$
   (c) $\displaystyle\int_0^2 \sqrt{4 - x^2}dx$
   (d) $\displaystyle\int_0^1 \frac{2x + 3}{\sqrt{4x + 6}}dx$

4. Consider the graph of $y = \tan(x)$, with the restricted domain $[-\pi/4, \pi/4]$. Set up, but **DO NOT** solve, a definite integral that equals the area bound by this graph and the straight line segment joining the two end points of the graph.

5. (5 points) Consider the solid generated by revolving the region bound by $y = \sqrt{x}$, $y = 1$, $y = 2$, and $x = 9$ about the x-axis.
   (a) Set up but **DO NOT** solve a formula involving definite integrals that equals the volume of this solid, using **cross sections**.
   (b) Set up but **DO NOT** solve a formula involving definite integral that equals the volume of this solid, using **cylindrical shells**.

6. Set up but **DO NOT** solve a definite integral that equals the perimeter of the ellipse given by $x^2 + \frac{y^2}{4} = 1$.

7. Find the surface area of the object obtained by rotating the finite length arc given by $y = \sqrt{2x - x^2}$, $0 \leq x \leq 2$ about the x-axis.

8. A water tank is shaped like an inverted cone. The height of the tank is 8 m, and the circular opening at the top of the tank has a radius of 2 m. The water in the tank is currently 4 m deep. How much work is required to pump the water to the level of the top of the tank and out of the tank?