**Math 1161: Written Homework 4** Name: _____ .# _____

Due October, 16, 2018 in recitation. TA: _____ Time: _____

*Instructions.* You may discuss this assignment with others, but you must submit your own write-up. Write clearly and legibly. All functions herein are real-valued functions of a single real variable. MVT abbreviates Mean Value Theorem.

1. (3 pts) Sometimes, in certain expressions, physicists and engineers replace an occurrence of $\sin(x)$ with $x$ provided that $x$ is a very small angle measured in radians.

   Justify this using the linear approximation to the function $f(x) = \sin(x)$ at a suitable value of $x$.

   Is this reasoning still applicable if $\theta$ is measured in **degrees**? Do the same linear approximation as above but this time with the angle measured in degrees and show the calculations to justify your answer.

2. (6 pts) Find an **equation** for the circle $\mathcal{C}$ with diameter on the $y$-axis and passing through the point $(1, 1)$ that encloses the least area. Show all your work and justify why this circle encloses the minimum possible area.

(continued on reverse)

3. (6 pts) Let $f$ be continuous on $[0, 2]$ and twice differentiable on $(0, 2)$. Suppose $f(0) = 0, f(1) = 1$, and $f(2) = 2$. Show that there is a number $c$ in $(0, 2)$ such that $f''(c) = 0$.(*Hint*: You can use the Mean Value Theorem on any function that satisfies the hypothesis of the theorem.)

4. (5 pts) Prove that $\dfrac{1}{x+1} < \ln(x+1) - \ln x < \dfrac{1}{x}$ for all $x > 0$. (*Hint*: Use the Mean Value Theorem.)