Math 1161: Written Homework 5 Name: ____________________ .# ________

Due October 30, 2018 in recitation. TA: ____________________ Time: ________

Instructions. You may discuss this assignment with others, but you must submit your own write-up. Write clearly and legibly. All functions herein are real-valued functions of a single real variable.

There are many times in engineering and the sciences when one cannot write an explicit elementary expression for an antiderivative of a function. In these situations, one must often settle for estimates of a definite integral that come from Riemann Sums or similar approximation techniques. It is often useful to be able to bound the size of the error of your estimate, and it is useful to know if your estimate is too high or too low.

Throughout this assignment, we suppose f is an integrable function on an interval $[a, b]$, and n is always a positive integer. We write L_n to represent the Left Regular Riemann Sum of f on $[a, b]$ with n equal subintervals, and we write R_n to represent the Right Regular Riemann Sum of f on $[a, b]$ with n equal subintervals.

1. (2 pts) If f is increasing (or non-decreasing), then $L_n \leq \int_a^b f(x)\,dx \leq R_n$.

   Give a "proof by picture" by sketching the same increasing function on the interval $[1, 4]$ twice, once illustrating the rectangles used to compute L_6 and once illustrating the rectangles used to compute R_6.

2. (2 pts) If f is decreasing (or non-increasing), then $R_n \leq \int_a^b f(x)\,dx \leq L_n$.

   Give a "proof by picture" by sketching the same decreasing function on the interval $[1, 4]$ twice, once illustrating the rectangles used to compute L_6 and once illustrating the rectangles used to compute R_6.

A function f is **monotone** on an interval I if f is non-decreasing or non-increasing on I.

The difference between the value of the definite integral and an approximation is the **error** for that approximation.

(continued on reverse)

3. Suppose f is a monotone function on $[a, b]$. Then, we can use the previous problems to get the following bounds on the size of the errors for L_n and R_n.

   (a) (5 pts) Prove that $\left|\int_a^b f(x)\,\mathrm{d}x - L_n\right| \leq |R_n - L_n|$.

   Similarly, $\left|\int_a^b f(x)\,\mathrm{d}x - R_n\right| \leq |R_n - L_n|$. (You do not have to show this.)

   (b) (5 pts) Show that $R_n - L_n = \left[f(b) - f(a)\right] \cdot \frac{b-a}{n}$.

4. Let $f(x) = 2^{\sqrt{x}}$.

   (a) (3 pts) Show that f is increasing on $[0, 4]$.

   (b) (3 pts) Consider the integral $\int_0^4 f(x)\,\mathrm{d}x$. Use Problem 3 to find n sufficiently large so that the errors for L_n and R_n are both less than 0.01.