

Math 1161: Written Homework 6

Name: _____ .# _____

Due November 13, 2018 in recitation.

TA: _____ Time: _____

Instructions. You may discuss this assignment with others, but you must submit your own write-up. Write clearly and legibly. All functions herein are real-valued functions of a single real variable.

1. (5 pts) Expressions like $\int_0^x x^2 dx$ are nonsense since the variable that you integrate over cannot occur in either of the limits of integration. However, there is nothing wrong with the integral expression $\int_0^x xf(t) dt$. Suppose that f is a continuous function on $(-\infty, \infty)$. Let $F(x) = \int_0^x xf(t) dt$. Compute $F'(x)$ (Hint: the answer is not $xf(x)$).

2. Define the **convolution** $f * g$ of the continuous functions f and g to be the function

$$F(x) = \int_0^x f(t)g(x-t) dt$$

- (a) (5 pts) Compute the convolution $f * g$ of the constant function $f(x) = 1$ with the constant function $g(x) = 1$. (Hint: your answer should be a **function**)

(continued on reverse)

(b) (5 pts) Compute the convolution $f * g$ of $f(x) = x$ with function $g(x) = x^2$. (Again, your answer should be a function)

(c) (5 pts) Perform the substitution $u = x - t$ in the definite integral $\int_0^x f(t)g(x - t) dt$. Use this substitution to conclude that for any continuous functions f and g

$$f * g = g * f$$