MATH 151.02 **GROUPWORK 2** October 6, 2009

NAMES: _____

**1.(3pts)** Let $m = f(A)$ be the minimum annual gross income, in thousands of dollars, needed to obtain a 30-year home mortgage loan of $A$ thousand dollars at an interest rate of 6 percent. What do the following quantities represent in terms of the income need for a loan?

(a) $f(100)$ is a minimum annual gross income, in thousands of dollars, that one needs in order to obtain a 30-year home mortgage loan of 100 thousand dollars at an interest rate of 6 percent.

(b) $f^{-1}(75)$ is the amount of thousand dollars that someone can get for a 30-year home mortgage loan (at an interest rate of 6 percent), if he/she has a minimum annual gross income of 75 thousand dollars

**2.(4pts)** A tree of height $y$ meters has, on average, $B$ branches, where $B = y - 1$. each branch has, on average, $n$ leaves, where $n = 2B^2 - B$. Find the average number of leaves on a tree as a function of height.

# of leaves on a $y$-meter tall tree $= f(y) = B \cdot n = B \cdot (2B^2 - B)$

($B$: # of branches; $n$: # of leaves on each branch)

So, $f(y) = (y-1)\left(2\cdot(y-1)^2 - (y-1)\right)$

**3.(4pts)** Solve for $t$ in terms of the other "variables".

(a) $Q = Q_0 a^{nt}$

$\log Q = \log Q_0 + \log a^{nt}$

$\log Q = \log Q_0 + nt \log a$

$nt \log a = \log Q - \log Q_0$

$t = \dfrac{\log Q - \log Q_0}{n \log a}$

b) $P_0 a^t = Q_0 b^t$

$\log P_0 + \log a^t = \log Q_0 + \log b^t$

$\log P_0 + t \log a = \log Q_0 + t \log b$

$t \log a - t \log b = \log Q_0 - \log P_0$

$t(\log a - \log b) = \log \dfrac{Q_0}{P_0}$

$t \log \dfrac{a}{b} = \log \dfrac{Q_0}{P_0}$

$t = \dfrac{\log \frac{Q_0}{P_0}}{\log \frac{a}{b}}$

**4.(3pts)** The exponential function $y(x) = Ce^{\alpha x}$ satisfies the conditions $y(0) = 2$ and $y(1) = 1$. Find the constants $C$ and $\alpha$.

$2 = y(0) = C \cdot e^{\alpha \cdot 0} = C \cdot 1 = C \Rightarrow C = 2$

$y(x) = 2e^{\alpha x}$

$1 = y(1) = 2e^{\alpha \cdot 1} = 2e^{\alpha}$

$e^{\alpha} = \frac{1}{2}$

$\alpha \ln e = \ln(\frac{1}{2})$

$\alpha \cdot 1 = \ln(\frac{1}{2})$ (since $\ln e = 1$)

$\alpha = \ln(\frac{1}{2})$

**5.(2pts)** What is the difference between $\sin x^2, \sin^2 x$, and $\sin(\sin x)$? Express each of the three as a composition of functions. (Note: $\sin^2 x$ is another way of writing $(\sin x)^2$.)

If we denote $f(x) = x^2$, $g(x) = \sin x$, then

$\sin x^2 = g(f(x))$

$\sin^2 x = f(g(x))$

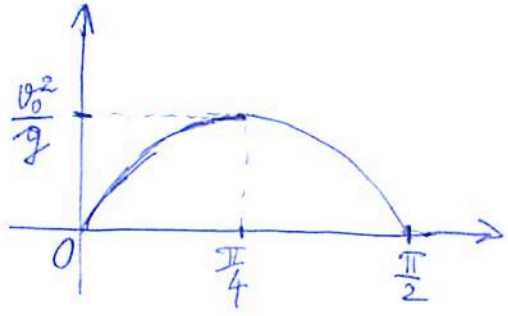
$\sin(\sin x) = g(g(x))$

**6.(4pts)** A baseball hit at an angle of $\theta$ to the horizontal with an initial velocity $v_0$ has a horizontal range $R$ given by

$$R = \frac{v_0^2}{g} \sin(2\theta)$$

where $g$ is the acceleration due to gravity.

(a) Sketch $R$ as a function of $\theta$ for $0 \le \theta \le \frac{\pi}{2}$.




(b) What angle gives the maximum range? What is the maximum range?

$\theta = \frac{\pi}{4}$ gives the maximum value of $R$.

The maximum value of $R$ is $R(\frac{\pi}{4}) = \frac{v_0^2}{g} \sin(2 \cdot \frac{\pi}{4}) = \frac{v_0^2}{g} \cdot 1 = \frac{v_0^2}{g}$