MATH 151.02 GROUPWORK3 October 20, 2009

NAMES: ____

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1.(4pts.) By using algebra, evaluate the following limits:

a) $\lim_{h\to 0}\frac{(4+h)^2-8}{h}$ DNE, since $(4+h)^2 - 8 \to 8$ $(h\to 0)$

and so, $\lim_{h\to 0+}\frac{(4+h)^2-8}{h}=\infty$, $\lim_{h\to 0-}\frac{(4+h)^2-8}{h}=-\infty$

b) $\lim_{h\to 0}\frac{(2+h)^3-8}{h}$

$=\lim_{h\to 0}\frac{\cancel{8}+12h+6h^2+h^3-\cancel{8}}{h}=\lim_{h\to 0}\frac{\cancel{h}(12+6h+h^2)}{\cancel{h}}=\lim_{h\to 0}(12+6h+h^2)=12$

c) $\lim_{h\to 0}\frac{(4+h)^2-(4-h)^2}{h}=\lim_{h\to 0}\frac{16+8h+h^2-(16-8h+h^2)}{h}=\lim_{h\to 0}\frac{\cancel{16}+8h+\cancel{h^2}-\cancel{16}+8h-\cancel{h^2}}{h}$

$=\lim_{h\to 0}\frac{16\cancel{h}}{\cancel{h}}=\lim_{h\to 0}16=16$

2.(4pts.) By using the definition of the derivative, find the derivatives of the given functions:

a) $f(x)=x^2+x+1$ at $x=2$

$f'(2)=\lim_{h\to 0}\frac{f(2+h)-f(2)}{h}=\lim_{h\to 0}\frac{(2+h)^2+(2+h)+1-(2^2+2+1)}{h}=\lim_{h\to 0}\frac{4+4h+h^2+3+h-7}{h}$

$=\lim_{h\to 0}\frac{h^2+5h}{h}=\lim_{h\to 0}\frac{h(h+5)}{h}=\lim_{h\to 0}(h+5)=5$

b) $f(x)=\frac{1}{x^2}$ at $x=3$

$f'(3)=\lim_{h\to 0}\frac{f(3+h)-f(3)}{h}=\lim_{h\to 0}\frac{\frac{1}{(3+h)^2}-\frac{1}{3^2}}{h}=\lim_{h\to 0}\frac{\frac{9-(3+h)^2}{9(3+h)^2}}{h}=\lim_{h\to 0}\frac{9-9-6h-h^2}{9h(3+h)^2}$

$=\lim_{h\to 0}\frac{\cancel{h}(-6-h)}{9\cancel{h}(3+h)^2}=\frac{-6}{9\cdot 3^2}=-\frac{2}{27}$

3.(4pts.) Decide whether the function $f(x)=(x+|x|)^2+1$ is differentiable at $x=0$.

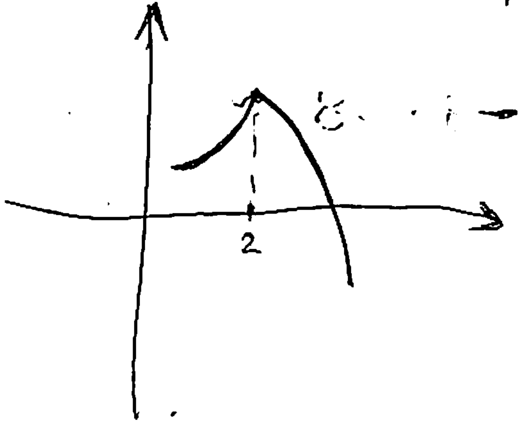
$\lim_{h\to 0}\frac{f(0+h)-f(0)}{h}=\lim_{h\to 0}\frac{h^2+2h|h|+|h|^2+1-((0+|0|)^2+1)}{h}=\lim_{h\to 0}\frac{2h^2+2h|h|}{h}$

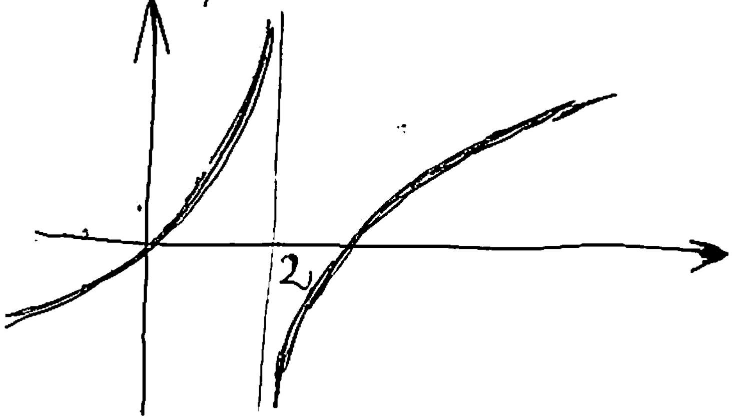
$=\lim_{h\to 0}\frac{2h(h+|h|)}{h}=\lim_{h\to 0}2(h+|h|)=2(0+|0|)=0$

4.(4pts.) Sketch the graph of the function with the following properties.

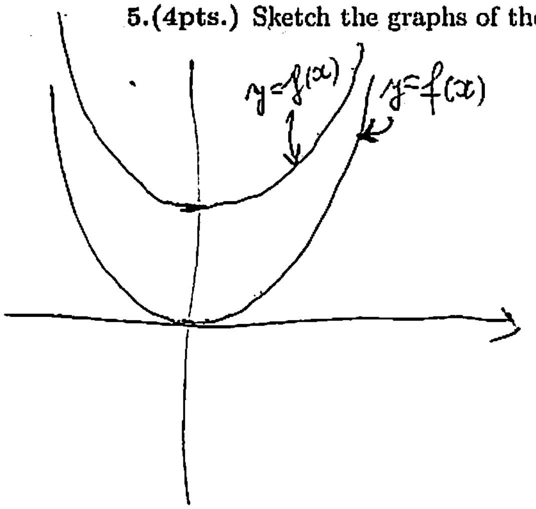
$f''(x) > 0$ for $x < 2$, $f''(x) < 0$ for $x > 2$ and $f'(2)$ is undefined.

Here are two possible examples





5.(4pts.) Sketch the graphs of the following functions: $f(x) = \frac{1}{2}x^2$, and $g(x) = f(x) + 3$.



a) What can you say about the slopes of the tangent lines to the two graphs at the points $x = 2, 3, 4$ and for any x_0?

For any point $x_0 \in \mathbb{R}$, $f'(x_0) = g'(x_0)$

b) Explain why adding a constant does not change the value of the slope at any point of the graph.

By adding a constant, we just make a vertical shift of the graph, so at every point, slope remains the same (corresponding tangent line just shifts vertically together with the graph)