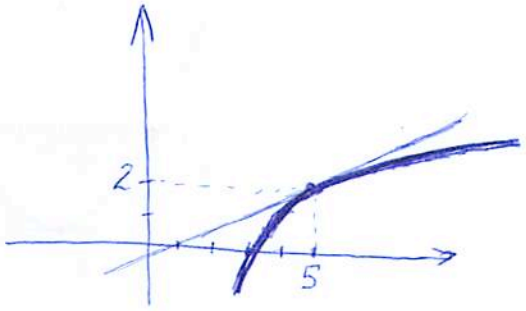
MATH 151.02 **GROUPWORK 4** October 27, 2009

NAMES: _____

1.(4pts.) A continous function defined for all x has the following properties: i) f is increasing; ii) f is concave down; iii) $f(5) = 2$; and iv) $f'(5) = \frac{1}{2}$.

a) Sketch a possible graph for f.



b) How many zeros does f have?

one

c) What can you say about locations of zeros of f ?

The zero of the function must be less than 5.

d) What is $\lim_{x\to-\infty} f(x)$?

$\lim_{x\to-\infty} f(x) = -\infty$

e) Is it possible that $f'(1) = 1$?

yes

f) Is it possible that $f'(1) = \frac{1}{4}$?

No, since it must be $f'(1) > f'(5) = \frac{1}{2}$ (because $f'' < 0$, and so f' is decreasing)

2.(2pts.) Find $\lim_{h\to 0} \frac{\sqrt{a+h} - \sqrt{a}}{h}$ $\cdot \frac{\sqrt{a+h}+\sqrt{a}}{\sqrt{a+h}+\sqrt{a}}$

$= \lim_{h\to 0} \frac{\sqrt{a+h}^2 - \sqrt{a}^2}{h(\sqrt{a+h}+\sqrt{a})} = \lim_{h\to 0} \frac{a+h-a}{h(\sqrt{a+h}+\sqrt{a})} = \lim_{h\to 0} \frac{h}{h(\sqrt{a+h}+\sqrt{a})} = \lim_{h\to 0} \frac{1}{\sqrt{a+h}+\sqrt{a}} = \frac{1}{2\sqrt{a}}$

3.(3pts.) a) Give an example of a function with $\lim_{x\to 5} f(x) = \infty$

$f(x) = \frac{1}{|x-5|}$ or $f(x) = \frac{1}{(x-5)^2}$

b) Give an example of a function with $\lim_{x\to 7} f(x) = -\infty$

$f(x) = \frac{-1}{|x-7|}$

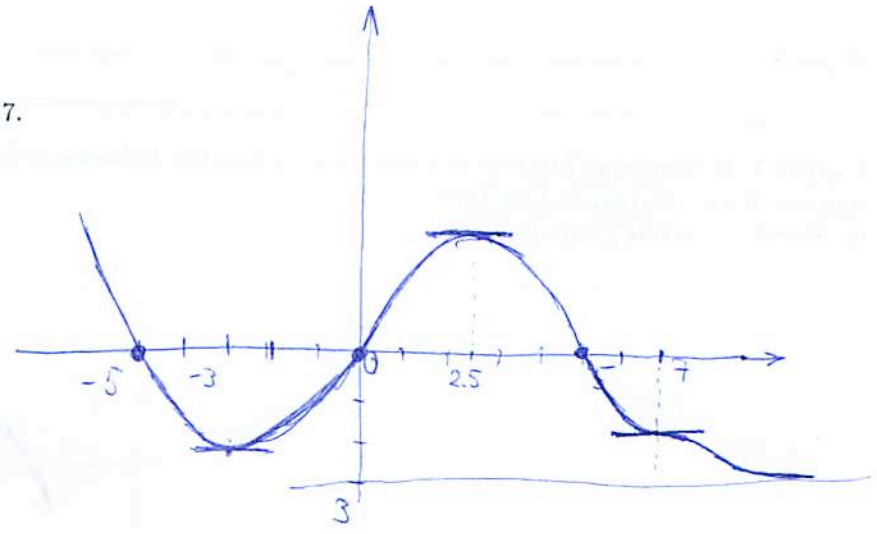
4.(4pts.) Given all of the following information about a function f, sketch its graph.

i) $f(x) = 0$ at $x = -5, 0, 5$

ii) $\lim_{x \to -\infty} f(x) = \infty$

iii) $\lim_{x \to \infty} f(x) = -3$

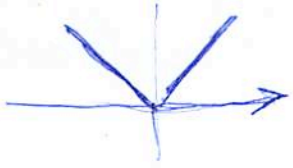
iv) $f'(x) = 0$ at $x = -3, 2.5, 7$.



5.(4pts.) Are the statements true or false? If a statement is true give an example to illustrating it. If a statement is false, give a counter example.

a) If a function is not differentiable then it is not continous.

FALSE



$y = |x|$

b) If a function is continous then it is differentiable.

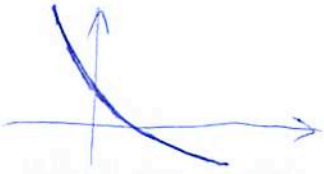
FALSE (statement "b) is the same as a), just said in other words

c) If f is concave up, then it is increasing.

FALSE

d) If f is decreasing then it is concave down.

FALSE



6.(3pts.) Find the derivative of

$f(x) = 7 + \pi + e + x^{\pi} + \frac{1}{x^5} + t^4 + x^{-3/4} + \frac{12}{\sqrt{x}} + x^{\frac{3}{2}}(4 + \sqrt{x})$ at any point x.

$x^{\frac{3}{2}} \cdot \sqrt{x} = x^{\frac{3}{2}} \cdot x^{\frac{1}{2}} = x^{\frac{4}{2}} = x^2$

$= 7 + \pi + e + x^{\pi} + x^{-5} + t^4 + x^{-\frac{3}{4}} + 12x^{-\frac{1}{2}} + 4x^{\frac{3}{2}} + x^2$

So, $f'(x) = 0 + 0 + 0 + \pi x^{\pi - 1} - 5x^{-6} + 0 - \frac{3}{4}x^{-\frac{7}{4}} - \frac{12}{2}x^{-\frac{3}{2}} + 4 \cdot \frac{3}{2}x^{\frac{1}{2}} + 2x$

i.e. $f'(x) = \pi x^{\pi - 1} - \frac{5}{x^6} - \frac{3}{4\sqrt[4]{x^7}} - \frac{6}{\sqrt{x^3}} + 6\sqrt{x} + 2x$