

NAMES: _____

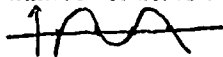
1.(4pts.) Assume that polynomial f has exactly two local maxima and one local minimum, and these are the only critical points of f :

a) Sketch a possible graph of f .



b) What is the largest number of zeros f could have?

4



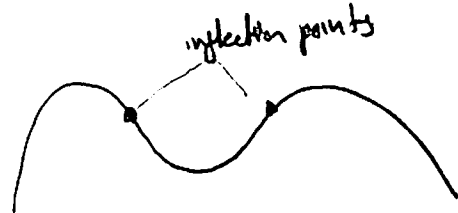
c) What is the least number of zeros f could have?

0



d) What is the least number of inflection points f could have?

2



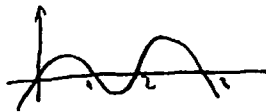
e) What is the smallest degree f could have?

4

there are 3 CP so degree of $f' \geq 3 \Rightarrow$ degree $f \geq 4$

f) Find a possible formula for f ?

$$-x(x-1)(x-2)(x-3)$$



2.(4pts.) Consider the function $f(x) = mx e^{-nx}$ for $m, n > 0$

a) Find the local maxima, local minima, and points of inflection.

$$f'(x) = m [e^{-nx} + x(-n)e^{-nx}]$$

$$f''(x) = m [(-n)e^{-nx} + (-n)x e^{-nx} + e^{-nx}(-n)]$$

$$\Rightarrow f''\left(\frac{1}{n}\right) = -mn e^{-1} (2-1) = -\frac{mn}{e} < 0$$

$$f'(x) = m e^{-nx} (1-nx)$$

$$= -mn e^{-nx} (1-nx+1)$$

$\Rightarrow \frac{1}{n}$ is local max

$$f'(x) = 0 \Rightarrow 1-nx = 0 \Rightarrow \boxed{x = \frac{1}{n}}$$

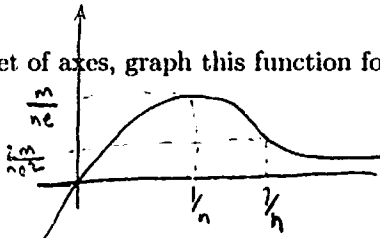
$$= -mn e^{-nx} (2-nx)$$

$$\Rightarrow f'' = 0 \Rightarrow 2-nx = 0 \Rightarrow \boxed{x = \frac{2}{n}}$$

inflection point

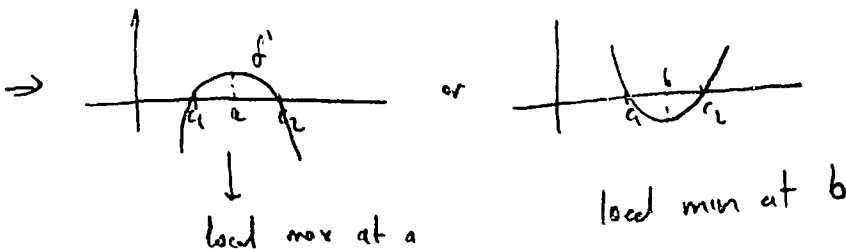
b) How does varying m, n affect the shape of the graph?

c) On one set of axes, graph this function for several values of m and n

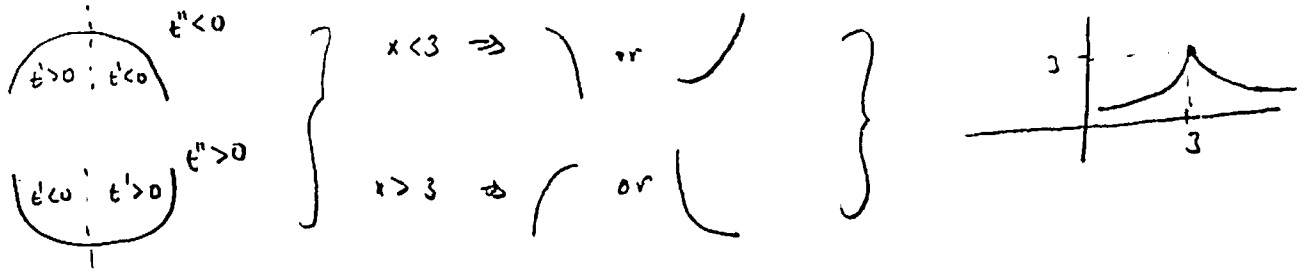


3.(3pts.) Show that if $f'''(x)$ is continuous and $f(x)$ has exactly two critical points, then $f'(x)$ has a local maximum or local minimum between the two critical points.

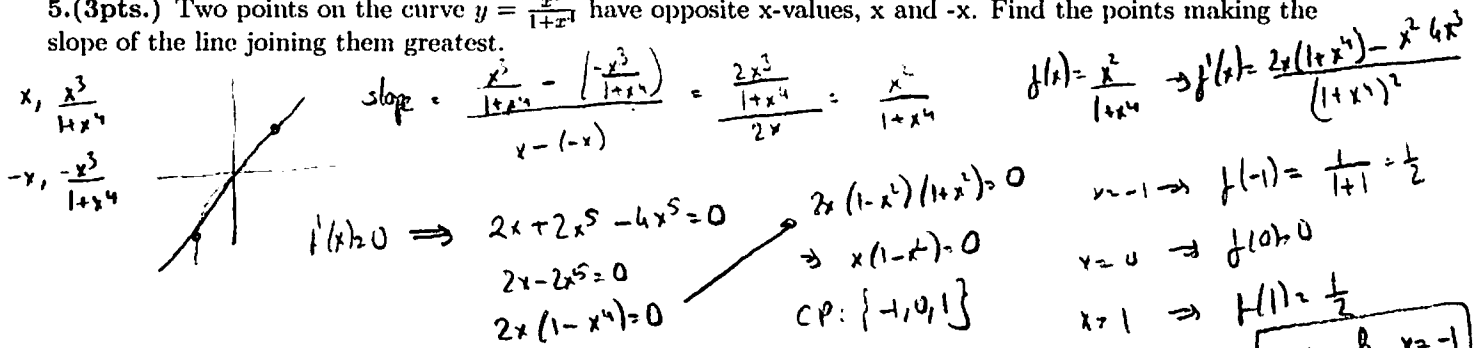
Let c_1, c_2 be the CP of $f(x) \Rightarrow f'(c_1) = f'(c_2) = 0$ [f''' continuous $\Rightarrow f'$ is diff.]



4. (3pts.) The function $y = t(x)$ is positive and continuous with a global maximum at the point (3,3). Graph $t(x)$ if $t'(x)$ and $t''(x)$ have the same sign for $x < 3$, but opposite signs for $x > 3$.



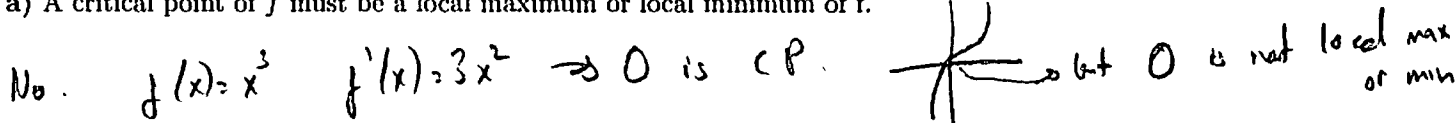
5. (3pts.) Two points on the curve $y = \frac{x^3}{1+x^4}$ have opposite x-values, x and $-x$. Find the points making the slope of the line joining them greatest.



6. (3pts.) Decide if the statements below are true or false. Give an explanation for your answer.

\Rightarrow $\boxed{x=1 \ \& \ x=-1}$
 $y = \frac{1}{2}$ $y = \frac{1}{2}$

a) A critical point of f must be a local maximum or local minimum of f .



b) Since the function $f(x) = \frac{1}{x}$ is continuous for $x > 0$ and the interval $(0,1)$ is bounded, f has a maximum on the interval $(0,1)$.



c) The Extreme Value Theorem says only continuous functions have global maxima and minima on every closed, bounded interval.

