MATH 151.02 **GROUPWORK 6** December 1, 2009

NAMES: ______________________________

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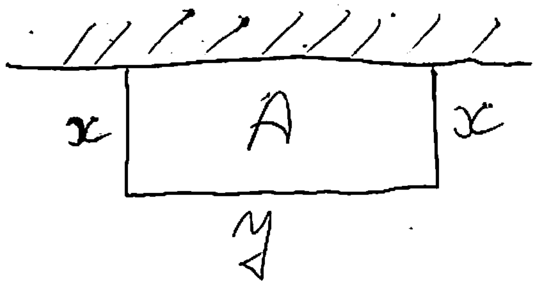
**1.(4pts.)** If you have 100 feet of fencing and want to enclose a rectangular area up against a long, straight wall, what is the largest area you can enclose?




$2x + y = 100$

$y = 100 - 2x$

$A = x \cdot y = x \cdot (100 - 2x)$

$A(x) = 100x - 2x^2$

$A'(x) = 100 - 4x \overset{set}{=} 0$

$x = 25$

$A(25) = 25 \cdot (100 - 2 \cdot 25)$

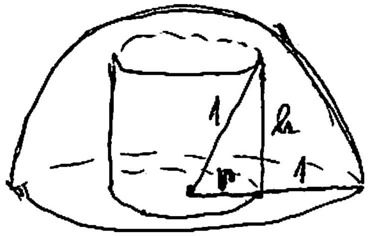
$\boxed{A(25) = 1250}$

$A'(x)$ $++$, $--$ at $25$

$A$ ↗ ↘

So at $x = 25$ is MAX

**2.(4pts.)** A hemisphere of radius 1 sits on a horizontal plane. A cylinder stands with its axis vertical, the center of its base at the center of the sphere, and its top circular rim touching the hemisphere. Find the radius and height of the cylinder of maximum volume.




$r^2 + h^2 = 1$

$r^2 = 1 - h^2$

$V = \pi r^2 h = \pi(1 - h^2)h$

$V(h) = \pi(h - h^3)$

$V'(h) = \pi(1 - 3h^2) \overset{set}{=} 0$

$1 - 3h^2 = 0$

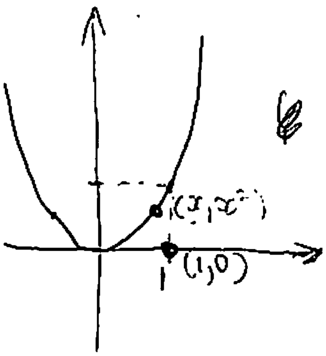
$h^2 = \frac{1}{3} \Rightarrow h = \pm\frac{1}{\sqrt{3}}$

$\Rightarrow h = \frac{1}{\sqrt{3}}$ $(h > 0)$

$r^2 = 1 - h^2 = 1 - \frac{1}{\sqrt{3}}^2 = \frac{2}{3}$

$$\boxed{r = \frac{2}{3} \quad h = \frac{1}{\sqrt{3}}}$$

3.(4pts.) Which point on the parabola $y = x^2$ is nearest to (1,0)? Find the coordinates to two decimals. [Hint: Minimize the square of the distance - this avoids square roots.]




$$d = \sqrt{(x-1)^2 + (x^2-0)^2}$$

$$\underbrace{d^2}_{f(x)} = (x-1)^2 + x^4$$

$$f'(x) = 2(x-1) + 4x^3 \overset{\text{set}}{=} 0$$

$$4x^3 + 2x - 2 = 0$$

Using calculator, $x \approx 0.5897$

So, the nearest pt. is $(0.59, 0.35)$

4.(4pts.) Find $\lim_{t \to 0+} \left(\frac{1}{t} - \frac{1}{e^t - 1}\right) = \lim_{t \to 0+} \frac{e^t - 1 - t}{t(e^t - 1)} \overset{\text{Lop. rule}}{=} \lim_{t \to 0+} \frac{e^t - 1}{e^t + te^t - 1}$

$$\overset{\text{Lop. rule}}{=} \lim_{t \to 0+} \frac{e^t}{e^t + e^t + te^t} = \frac{e^0}{e^0 + e^0 + 0 \cdot e^0} = \frac{1}{1+1+0} = \frac{1}{2}$$

5.(4pts.) Find $\lim_{x \to \infty} (1+x)^{1/x} = \lim_{x \to \infty} e^{\ln(1+x)^{1/x}} = \lim_{x \to \infty} e^{\frac{1}{x} \ln(1+x)}$

$$= \lim_{x \to \infty} e^{\frac{\ln(1+x)}{x}} = e^{\lim_{x \to \infty} \frac{\ln(1+x)}{x}} \overset{\text{Lop. rule}}{=} e^{\lim_{x \to \infty} \frac{\frac{1}{1+x}}{1}}$$

$$= e^{\lim_{x \to \infty} \frac{1}{1+x}} = e^0 = 1$$