1.2.2 The graph is concave down.

1.2.3 The graph is neither concave up nor concave down.

1.2.6 $P = 7.7(0.92)^t$
a) The initial quantity is $P(0) = 7.7 * (0.92)^0 = 7.7 * 1 = 7.7$
b) The growth rate is $a - 1 = 0.92 - 1 = -0.08$, so it it actually a decay rate of 8%.
c) This function is presented in the form $P = P_0 a^t$, therefore it is *not* assumed to be continuous.

1.2.8 $P = 15 * e^{-0.06t}$
a) The initial quantity is $P(0) = 15 * e^{-0.06*0} = 15 * e^0 = 15 * 1 = 15$
b) The growth rate is $r = -0.06$, so it is a decay rate of 6%.
c) This function is presented in the form $P = P_0 e^{rt}$, therefore it is assumed to be continuous.

1.2.15 A town has a population of 1000 people at time $t = 0$. In each of the following cases, write a formula for the population, P, of the town as a function of the year t.
a) The population increases by 50 people a year. $P = 1000 + 50t$
b) The population increases by 5 percent a year. $P = 1000(1.05)^t$

1.2.20 When a new product is advertised, more and more people try it. However the rate at which new people try it slows down as time goes on.
a) Graph the total number of people who have tried such a product against time.
Your graph should resemble Figure 1.24 on page 15 of your text, with Q representing the total number of people who have tried the product.
b) What do you know about the concavity of the graph?
This graph should be concave down.

1.2.27 When the Olympic Games were held outside Mexico City in 1968, there was much discussion about the effect the high altitude (7340 feet) would have on the athletes. Assuming air pressure decays exponentially by 0.4 percent every 100 feet, by what percentage is air pressure reduced by moving from sea level to Mexico City?

It is reasonable to assume that we can measure air pressure continuously, and therefore we can use the continuous version of exponential decay. If we let P_0 the air pressure at sea level, then at a height h measured in hundreds of feet (since our decay rate was given for every 100 feet),

$$P(h) = P_0 e^{-0.004h}.$$

Thus the air pressure in Mexico City would be

$$P(73.4) = P_0 e^{-0.004*73.4} \approx 0.7456 P_0.$$

Therefore the air pressure has been reduced by approximately 25.44% in moving from sea level to Mexico City.

1.2.30 In the early 1960s, radioactive strontium-90 was released during atmospheric testing of nuclear weapons an got into the bones of people alive at the time. If the half life of strontium-90 is 29 years, what fraction of the strontium-90 absorbed in 1960 remained in peoples' bones in 1990?

If we assume that radioactive decay is a continuous process (which is a reasonable assumption) the amount, Q, of strontium-90 remaining in a person's bones can be expressed as

$$Q = Q_0 e^{rt},$$

where Q_0 is the initial amount of strontium-90 present in 1960, r the rate at which strontium-90 decays, an t the number of years since 1960. Since the half life is 29 years, we know that

$$\frac{1}{2}Q_0 = Q(29) = Q_0 e^{29r}$$

Dividing the left and right sides of this equation by Q_0, and then applying the natural log to both sides we find

$$\begin{aligned} \frac{1}{2} &= e^{29r} \\ \ln\frac{1}{2} &= \ln e^{29r} \\ \ln 2^{-1} &= 29r \ln e \\ \ln 2^{-1} &= 29r \\ \frac{\ln 2^{-1}}{29} &= r. \end{aligned}$$

Thus

$$e^{rt} = (e^r)^t = (e^{\frac{\ln 2^{-1}}{29}})^t = (e^{\ln 2^{-1}})^{\frac{t}{29}} = (2^{-1})^{\frac{t}{29}} = 2^{\frac{-t}{29}}.$$

So our function for the amount of strontium-90 in a person's bones at a time t years after 1960 is

$$Q = Q_0(2)^{\frac{-t}{29}}.$$

Therefore, in 1990 a person will have

$$Q(30) = Q_0(2)^{\frac{-30}{29}} \approx 0.488Q_0,$$

or approximately 48.8% of the initial amount left in their bones.