1.4.3 $5e^{\ln A^2} = 5A^2$

1.4.5 $\ln(1/e) + \ln(AB) = \ln 1 - \ln e + \ln a + \ln B = 0 - 1 + \ln A + \ln B = \ln A + \ln B - 1$

1.4.11

$$\begin{aligned} 7 &= 5e^{0.2x} \\ 7/5 &= e^{0.2x} \\ \ln 7/5 &= 0.2x \\ \frac{\ln 7 - \ln 5}{0.2} &= x \end{aligned}$$

1.4.12

$$\begin{aligned} 2^x &= e^{x+1} \\ x\ln 2 &= x+1 \\ x\ln 2 - x &= 1 \\ x(\ln 2 - 1) &= 1 \\ x &= \frac{1}{\ln 2 - 1} \end{aligned}$$

1.4.20

$$\begin{aligned} P &= P_0 a^t \\ \ln P &= \ln P_0 + t\ln a \\ \frac{\ln P - \ln P_0}{\ln a} &= t \end{aligned}$$

1.4.26

$$\begin{aligned} P &= 10(1.7)^t \\ &= 10(e^{\ln 1.7})^t \\ &= 10e^{t\ln 1.7} \end{aligned}$$

1.4.31 Find the inverse function of $f(t) = 1 + \ln t$

$$\begin{aligned} t &= 1 + \ln x \\ t - 1 &= \ln x \\ e^{t-1} &= x \end{aligned}$$

So $f^{-1}(t) = e^{t-1}$

1.4.40 The population of a region is growing exponentially. there were 40,000,000 people in 1990 ($t = 0$) and 56,000,000 in 2000.
a) Find an expression for the population at any time t in years.
We'll start with the formula $P = P_0 a^t$, measures in millions. Since $P(0) = 40, P_0 = 40$. So we "upgrade" our formula to $P = 40a^t$. We can solve for a using the second piece of information given.

$$\begin{aligned} 56 = P(10) &= 40a^{10} \\ 8/5 = 56/40 &= a^{10} \\ (8/5)^{\frac{1}{10}} &= a \end{aligned}$$

So our equation is $P = 40(8/5)^{\frac{t}{10}}$.

b) The population in 2010 would be

$$P(20) = 40(8/5)^{20/10} = 40(8/5)^2 = 40 * \frac{8^2}{5^2} = 8^3/5,$$

in millions.

c) The doubling time, t_D can be found by noticing that $P(t_D) = 2P_0 = 80$. So

$$\begin{aligned} 80 = P(t_D) &= 40(8/5)^{\frac{t_D}{10}} \\ 2 &= (8/5)^{\frac{t_D}{10}} \\ \ln 2 &= \frac{t_D}{10} \ln(8/5) \\ \frac{\ln 2}{\ln 8 - \ln 5} &= t_D/10 \\ \frac{10 \ln 2}{\ln 8 - \ln 5} &= t_D. \end{aligned}$$