1.8

$\boxed{22}$ 24, 42, 46, 58

$\lim_{x\to c^+} f(x) = L$ : for every $\varepsilon > 0$ there is a $\delta > 0$ st

$$x - c < \delta \Rightarrow |f(x) - L| < \varepsilon$$

(since $x > c$ we do not need absolute value)

$\boxed{24}$ $\lim_{x\to\infty} f(x) = L$ : for every $\varepsilon > 0$ there is $N$ st

$$x \geq N \Rightarrow |f(x) - L| < \varepsilon$$

$\boxed{42}$ $$\lim_{x\to\infty} \frac{x^4+3x}{x^4+2x^5} = \lim_{x\to\infty} \frac{\frac{x^4}{x^5}+\frac{3x}{x^5}}{\frac{x^4}{x^5}+\frac{2x^5}{x^5}} = \lim_{x\to\infty} \frac{\frac{1}{x}+\frac{3}{x^4}}{\frac{1}{x}+2} = \frac{0+0}{0+2} = 0$$

$\boxed{46}$ $\lim_{x\to 4} \frac{x^2-k^2}{x-4}$ if $x=4 \Rightarrow$ denomiter $=0 \Rightarrow$ to make limit exists we need to make also nomerator $=0$

$$\Rightarrow 4^2 - k^2 = 0$$

$$\Rightarrow \boxed{k = \mp 4}$$

$k = \mp 4 \to$ $$\lim_{x\to 4} \frac{x^2-k^2}{x-4} = \lim_{x\to 4} \frac{x^2-16}{x-4} = \lim_{x\to 4} \frac{\cancel{(x-4)}(x+4)}{\cancel{(x-4)}} = \lim_{x\to 4}(x+4) = \boxed{8}$$

$\boxed{58}$ ⓐ $b=0 \Rightarrow \lim_{x\to c} b f(x) = \lim_{x\to c} 0 = 0 = 0 \lim_{x\to c} f(x) = b \lim_{x\to c} f(x)$

ⓒ $\lim_{x\to c} f(x) = L \Rightarrow$ by definition for $\frac{\varepsilon}{|b|}$, there exits $\delta$ st

$$|x-c| < \delta \Rightarrow |f(x) - L| < \frac{\varepsilon}{|b|}$$

$$\xRightarrow{(by\ b)} |bf(x) - bL| < \varepsilon$$

ⓑ (if $z > 0$ & $x < y \Rightarrow zx < zy$)

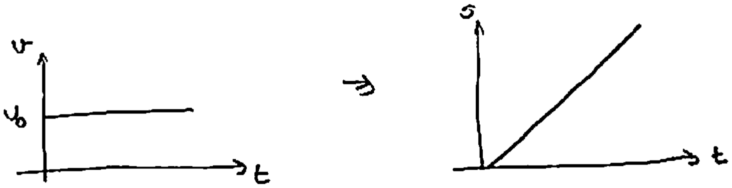
$$|b| > 0 \Rightarrow |b| \, |f(x) - L| < |b| \frac{\varepsilon}{|b|}$$

$$|bf(x) - bL| < \varepsilon$$

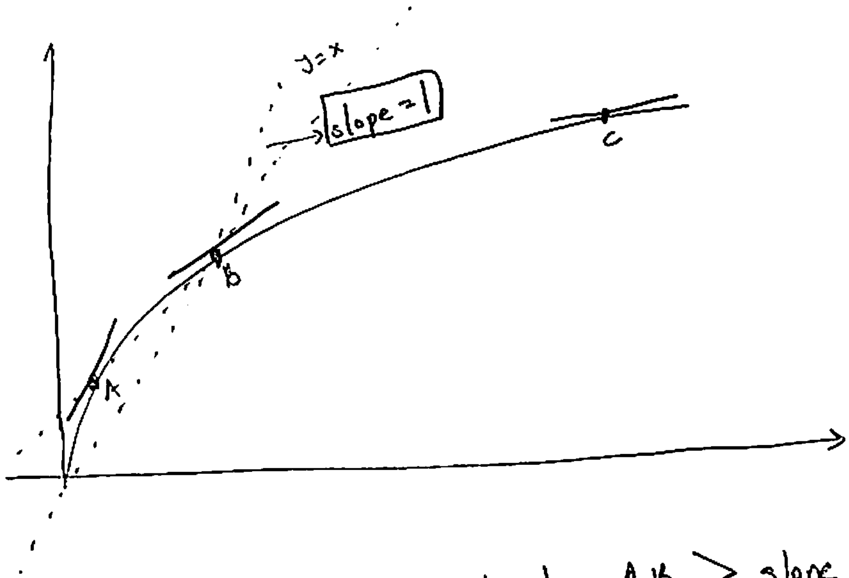
2.1 14, 17, 20, 26

#14 plug 0.1, 0.01, 0.001 into $\frac{e^{1+h}-e}{h}$

#17




#20




slope at A > 1 > slope of the line AB > slope at B > slope at C > 0

#26
$$\lim_{h\to 0}\frac{(1+h)^3-1}{h} = \lim_{h\to 0}\frac{\cancel{1}+3h+3h^2+h^3\cancel{-1}}{h}$$
$$= \lim_{h\to 0}\frac{\cancel{h}(3+3h+h^2)}{\cancel{h}}$$
$$= \lim_{h\to 0} 3+3h+h^2$$
$$\boxed{=3}$$

2.6 10, 15

#1 a) at 1

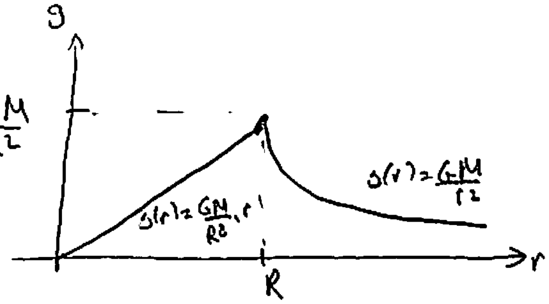
b) at 1, 2, 3

#10 ⓐ $r < R \Longrightarrow g(r) = \frac{GM}{R^3} \cdot r \longrightarrow$ line with slope $\frac{GM}{R^3}$

$r \geq R \Longrightarrow g(r) = GM \cdot \frac{1}{r^2}$

at R $\left(\frac{GM}{R^3} \cdot r\right)_{r=R} = \frac{GM}{R^3} \cdot R = \frac{GM}{R^2}$

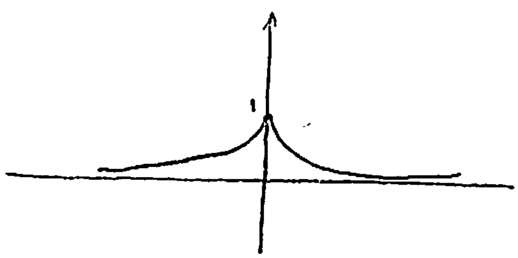
$\left(GM \frac{1}{r^2}\right)_{r=R} = \frac{GM}{R^2}$




ⓑ g is continous, no jump or hole.

ⓒ not differentiable at R. (sharp corner)

#15 $\varnothing(x) = \begin{cases} \sqrt{x^2+1} - x & \text{for } x \geq 0 \\ \sqrt{x^2+1} + x & \text{for } x < 0 \end{cases}$




$\Longrightarrow$ a) continous at 0

b) not diff. sharp corner.