

SHOW ALL YOUR WORK! GOOD LUCK!

NAME: _____

1. (5pts.) By using the definition of the derivative, find the derivative of $f(x) = 5x^2 + 9x + 17$ at point $x = 3$.

$$\begin{aligned}
 f'(3) &= \lim_{h \rightarrow 0} \frac{f(3+h) - f(3)}{h} = \lim_{h \rightarrow 0} \frac{5(3+h)^2 + 9(3+h) + 17 - (5 \cdot 3^2 + 9 \cdot 3 + 17)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{5(9 + 6h + h^2) + 27 + 9h + 17 - (45 + 27 + 17)}{h} = \lim_{h \rightarrow 0} \frac{45 + 30h + 5h^2 + 27 + 9h + 17 - 45 - 27 - 17}{h} \\
 &= \lim_{h \rightarrow 0} \frac{39h + 5h^2}{h} = \lim_{h \rightarrow 0} \frac{h(39 + 5h)}{h} = \lim_{h \rightarrow 0} (39 + 5h) = 39 + 5 \cdot 0 = 39
 \end{aligned}$$

2. (5pts.) Find the following derivatives of the function $f(x) = |x|$, if they exist. If not, explain why a given derivative doesn't exist.

a) $f'(0) = \lim_{h \rightarrow 0} \frac{|0+h| - |0|}{h} = \lim_{h \rightarrow 0} \frac{|h|}{h}$ This limit does not exist (DNE)

because $\lim_{h \rightarrow 0^+} \frac{|h|}{h} = \lim_{h \rightarrow 0^+} \frac{h}{h} = \lim_{h \rightarrow 0^+} 1 = 1$

while $\lim_{h \rightarrow 0^-} \frac{|h|}{h} = \lim_{h \rightarrow 0^-} \frac{-h}{h} = \lim_{h \rightarrow 0^-} (-1) = -1$

Since R-H limit and L-H limit are not the same, $f'(0)$ DNE

b) $f'(3) = \lim_{h \rightarrow 0} \frac{f(3+h) - f(3)}{h} = \lim_{h \rightarrow 0} \frac{|3+h| - |3|}{h} = \lim_{h \rightarrow 0} \frac{|3+h| - 3}{h}$

$3+h > 0$ for h close to 0

$$= \lim_{h \rightarrow 0} \frac{3+h-3}{h} = \lim_{h \rightarrow 0} \frac{h}{h} = \lim_{h \rightarrow 0} 1 = 1$$

So, $f'(3) = 1$