MATH 151.02 **QUIZ 5** November 19, 2009

SHOW ALL YOUR WORK! GOOD LUCK! NAME: ____________________

1.(5pts.) Let $c > 0$ be constant.

(a) Find all critical points of $f(x) = \dfrac{c}{x^2} + x$.

Solution: x_c is a critical point of f if x_c is in the domain of f and either $f'(x)$ is undefined at x_c or $f'(x_c) = 0$. $f'(x) = -2c/x^3 + 1$ is undefined only at $x = 0$, which is not in the domain of f. So our only critical point comes from the solution of $-2c/x_c^3 + 1 = 0$, which is $x_c = \sqrt[3]{2c}$.

(b) Use the second derivative test to show that the graph has a local minimum.

Solution: $f''(x) = 6c/x^4$. $x^4 > 0$ for all x in the domain of f, and, since $c > 0$, $6c > 0$ as well. Therefore $f''(x) > 0$ for all x in the domain of f. Specifically, $f''(x_c) > 0$. The second derivative test says that $f'(x_c) = 0$ and $f''(x_c) > 0$ implies that the point $(x_c, f(x_c))$ is a local min.

2.(5pts.) Assume $L, A, k > 0$ are constant. Let $g(t) = L/(1 + Ae^{-kt})$.

(a) Show that g is increasing on $[0, +\infty)$.

Solution: $g'(t) = LAke^{-kt}/(1 + Ae^{-kt})^2$. $LAk > 0$ by assumption, and $e^{-kt}, (1 + Ae^{-kt})^2$ are both positive for all t. So $g' > 0$ everywhere, specifically on $[0, \infty)$. Therefore g is increasing on $[0, \infty)$.

(b) For $0 \leq a < b < \infty$, where on $[a, b]$ does g take its maximum value? (Justify your answer).

Solution: For every $x \in [a, b), x < b$. Per the definition of an increasing function, $x < b$ implies $g(x) < g(b)$. So for all $x \in [a, b)$, $g(x) < g(b)$. So $g(b)$ is the maximum value of g on $[a, b]$, i.e. g takes its maximum value at b.