Math 151A Final Exam
December 8, 2008, 5:30 - 7:18 PM

Name: _____

Recitation Instructor and time: _____

This exam has 9 questions, for a total of 80 points on 11 pages.

Please read the problems carefully. Please show your work. Your solutions must be supported by computations and/or explanations: no points will be given for answers that are not accompanied by supporting work.

NO CALCULATORS.

Problem #	Points	Score
1	10	
2	7	
3	5	
4	8	
5	10	
6	8	
7	12	
8	10	
9	10	
Total	80	

151A Final Exam, December 8, 2008

1. (10 points) For each of the following, decide whether the statement is True or False. Circle T for *True* or F for *False*. (1 point for a correct answer, 1/2 point for no answer, 0 points for the wrong answer.)

   T F The graph of $f(x-2)$ is the graph of $f(x)$ shifted to the left by 2 units.

   T F The graph of $f(x-2)$ is the graph of $f(x)$ shifted up by 2 units.

   T F The amplitude of $\sin(3x)$ is 3.

   T F The function $f(x) = |x|$ is continuous at $x = 0$.

   T F Every polynomial $p(x)$ has a real root (i.e., $p(x) = 0$ has a solution for some $x \in (-\infty, \infty)$).

   T F If $\lim_{x\to a} |f(x)| = 1$, then $\lim_{x\to a} f(x) = 1$.

   T F If $\lim_{x\to a} |f(x)| = 0$, then $\lim_{x\to a} f(x) = 0$.

   T F The derivative of the position function is acceleration.

   T F Suppose $f(x)$ is differentiable on $[-1, 1]$, $f(-1) = 1$ and $f(1) = 2$. Then there is a point c in $(-1, 1)$ such that $f(c) = 0$.

   T F Suppose $f(x)$ is differentiable on $[-1, 1]$, $f(-1) = 1$ and $f(1) = 2$. Then there is a point c in $(-1, 1)$ such that $f'(c) = \frac{1}{2}$.

2. (a) (3 points) Give the formal definition, as a limit, of the *derivative* of a function $f(x)$.

(b) (4 points) Suppose $f(x) = x^2$. Using only your answer to part a), show algebraically that $f'(x) = 2x$.

3. (5 points) Find the tangent line to the curve, $y = \ln x$, at the point $(x, y) = (e^2, 2)$.

4. Find dy/dx if

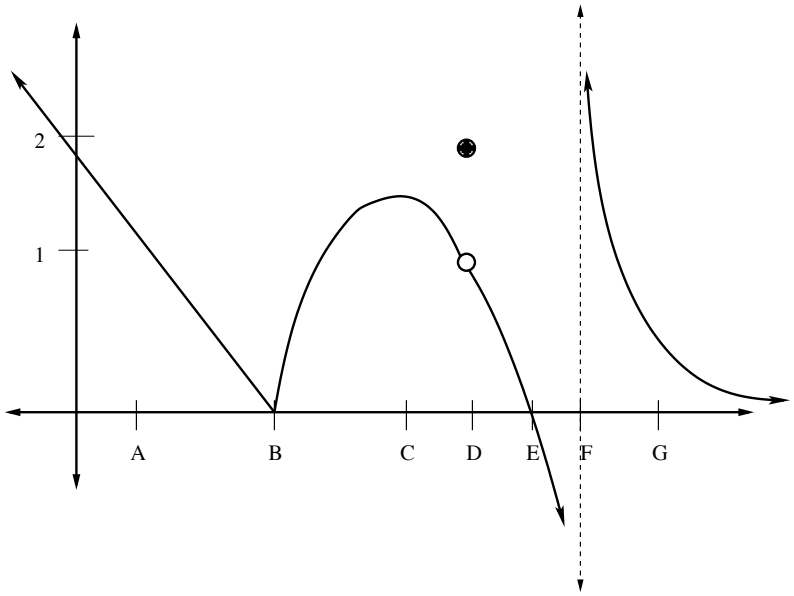
   (a) (4 points) $y = \arctan(x^2 + 2)$

   (b) (4 points) $y = \sqrt[3]{\sin x}$

5. (a) (6 points) Use implicit differentiation to find dy/dx when (x, y) lies on the curve $x^3 - y^3 = 4xy - 1$.

(b) (4 points) Find the equation of the tangent line to the curve at the point $(2, 1)$.

6. (8 points) This is a graph of the function $y = f(x)$.



(a) At which values of x is $f(x)$ not continuous?

(b) At which values of x is $f(x)$ not differentiable?

(c) Does $\lim_{x \to D} f(x)$ exist? If so, what is its value?

(d) Does $\lim_{x \to F} f(x)$ exist? If so, what is its value?

7. (12 points) Let $f(x) = \dfrac{x^3}{3} + \dfrac{x^2}{2} - 6x + 4$.

   (a) Find the y-intercept of $f(x)$.

   Answer: $y =$_____

   (b) Determine the interval(s) on which $f(x)$ is increasing and on which $f(x)$ is decreasing.

   Answer: increasing _____
   Answer: decreasing _____

   (c) Find the coordinates of each local maximum and local minimum of $f(x)$.

   Answer: local maxima, $(x, y) =$ _____
   Answer: local minima, $(x, y) =$ _____

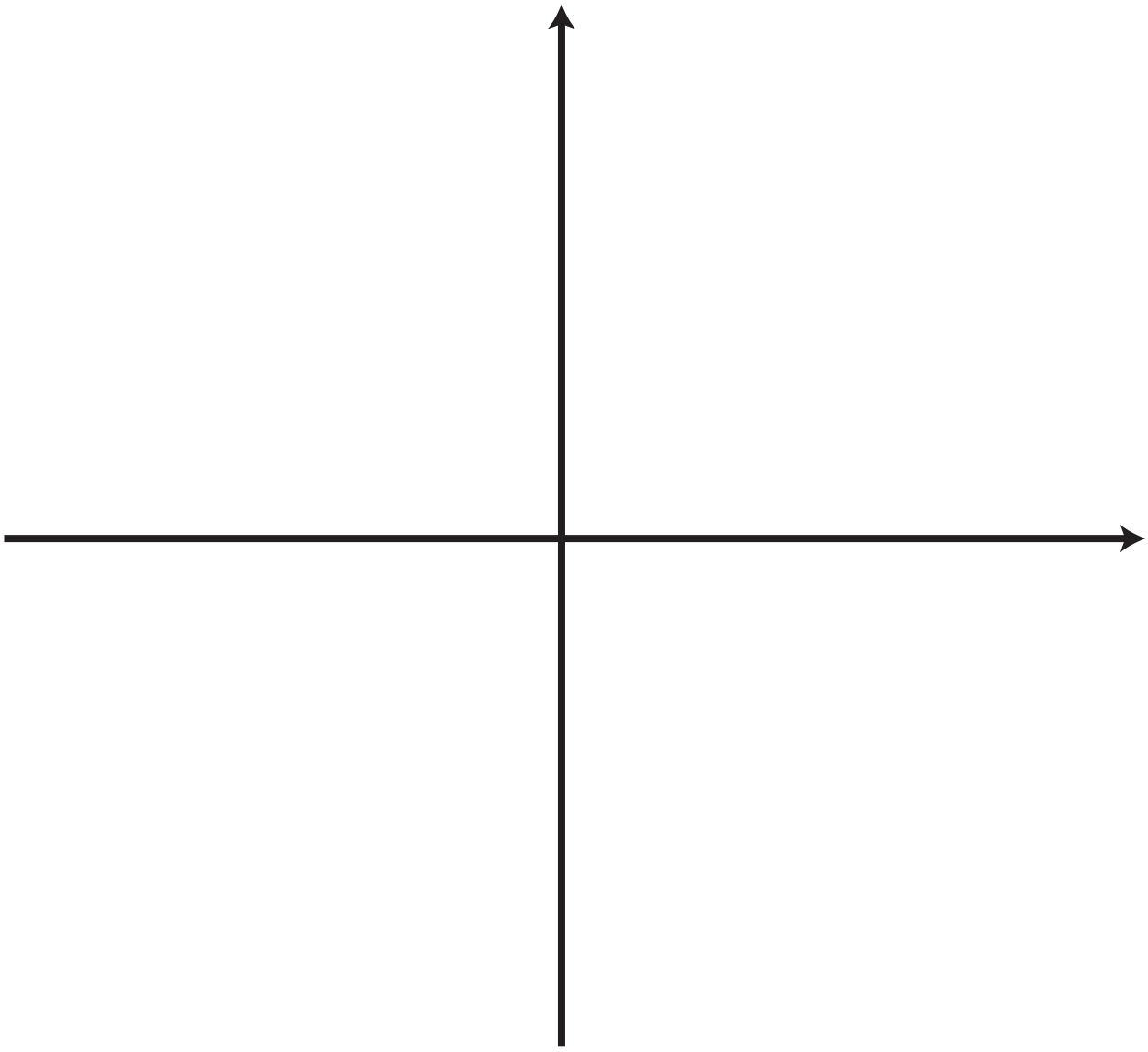
   (d) Determine the interval(s) on which $f(x)$ is concave up and on which $f(x)$ is concave down.

   Answer: concave up _____
   Answer: concave down _____

(e) Find the coordinates of each inflection point of $f(x)$.

Answer: inflection point(s), $(x, y) =$_____

(f) Using (a)-(e), sketch a graph of $f(x)$ on the axes below.
(Be sure to label all local extrema and inflection points.)



8. (10 points) A gas station stands at the intersection of a north-south road and an east-west road. A police car is traveling toward the gas station from the east, chasing a stolen truck which is traveling north away from the gas station. The speed of the police car is 100 mph at the moment it is 3 miles from the gas station. At the same time, the truck is 4 miles from the gas station going 80 mph. At this moment, is the distance between the car and truck increasing or decreasing? How fast? (Distance is measured along a straight line joining the car to the truck.)

9. (10 points) Find the area of the largest rectangle with base on the x-axis and upper vertices on the parabola $y = 27 - x^2$.