

**Mathematics 152.02**  
**Review for Midterm 2**

1. Solve the following initial value problem:

$$f'(x) = \frac{x^3 + 4x + 1}{x^2 + 3} \quad \text{where} \quad f(0) = 4.$$

2. Use integration by parts to show that for any constants  $a$  and  $b$  which are not both zero,

$$\int e^{ax} \cos(bx) dx = \frac{1}{a^2 + b^2} e^{ax} (a \cos(bx) + b \sin(bx)) + C.$$

3. Find the derivative  $f'(x)$  of the function  $f(x) = \int_{\sqrt{x}}^{x^3} \frac{t}{1 + e^t} dt$ .

4. Let LEFT( $n$ ), RIGHT( $n$ ), MID( $n$ ), TRAP( $n$ ), and SIMP( $n$ ) denote the usual approximations to the integral  $\int_0^{1/2} \sin(x^2) dx$ . Are the following statements true or false? Give reasons.

- (a)  $2 \cdot \text{LEFT}(16) - \text{LEFT}(8) = \text{MID}(8)$
- (b)  $\text{LEFT}(300) < \text{RIGHT}(300)$
- (c)  $\text{MID}(450) < \text{TRAP}(450)$
- (d) to five decimal places,  $\text{SIMP}(2) = 0.04148$

5. Find the area enclosed by the curve  $y = \sqrt{4 - x^2}$ , the  $x$ -axis, the  $y$ -axis and the line  $x = 1$ .

6. The acceleration of a particle moving in a long thin tube satisfies the equation

$$a(t) = (1 + t)^2$$

for time  $t \geq 0$ . Find a formula for the position of the particle if it starts at time  $t = 0$  at rest at distance 0 from the center of the tube.

7. Define a function by the equation  $l(x) = \int_1^x \frac{1}{t} dt$ . Just using general properties of integrals (i.e., without assuming any properties of  $\ln(x)$ ) show that  $l(ab) = l(a) + l(b)$ . (Hint: use additivity of the integral and a substitution.)

8. Does the improper integral  $\int_1^\infty t^2 e^{-t} dt$  converge or diverge? If it converges, find its value.

9. The function  $f(x)$  is defined by  $f(x) = \int_0^x \sin(1 - t^2) dt$ . Are the following statements true or false?

- (a)  $f(0.5)$  is positive
- (b)  $f(x)$  has a maximum at  $x = 1$
- (c)  $f(x)$  is decreasing near  $x = 2$
- (d)  $f(x)$  is concave down near  $x = 3$