Mathematics 152.02
Review for Final Exam

1. Suppose that the function $f(t)$ represents the price of a barrel of oil in dollars at time t, measured in days, where $f(0)$ is the price at midnight on December 31, 2005. In words, what is represented by $\frac{1}{90}\int_0^{90} f(t)\,dt$?

2. Suppose that $f(x)$ is an odd function, $g(x)$ is an even function, and that the following values are known:
$$\int_0^3 f(x)\,dx = 2 \quad \int_3^5 f(x)\,dx = 1 \quad \int_0^3 g(x)\,dx = 4 \quad \int_3^5 g(x)\,dx = 1.$$
Find $\int_{-3}^5 f(x) + g(x)\,dx$.

3. Find the solution to the initial value problem
$$f'(t) = \frac{t^3+1}{t^3-4t} \qquad f(1) = 2$$

4. Show that the function $f(x) = \frac{1}{x}$ is concave up for $x > 0$. Without using your calculator, show that $\frac{2}{3} \leq \int_1^2 \frac{1}{x}\,dx \leq \frac{3}{4}$. (Hint: compute MID(1) and TRAP(1).) Find the SIMP(1) estimate for this integral.

5. The function $f(t)$ is known to be decreasing and concave down for $0 \leq t \leq 2$. Put the following estimates for $\int_0^2 f(t)\,dt$ in increasing order: LEFT(10), LEFT(20), MID(20), RIGHT(10), RIGHT(20), SIMP(20), TRAP(20).

6. For each of the following improper integrals, say whether it converges or diverges. If it converges, find its value.
$$\int_0^\infty \frac{1}{1+x^2}\,dx \qquad \int_2^\infty \frac{1}{x}\,dx \qquad \int_{0.5}^2 \frac{1}{1-x}\,dx \qquad \int_1^\infty \frac{1}{x^3}\,dx$$

7. A solid has its base in the (x, y)-plane. The base of the solid is the region between the curve $y = 4 - x^2$ and the x-axis. Each cross-section of the solid perpendicular to the x-axis is a square. Find the volume of the solid.

8. Find the volume of the donut obtained by rotating a circle of radius 1 and center $(0, 2)$ around the x-axis.

9. Suppose that $g(x)$ is a function such that $g(x) \geq 0$ and $g'(x) \geq 1$ for all $x \geq 0$. Define a function $f(x)$ by the formula $f(x) = \int_0^x \sqrt{(g'(t))^2 - 1}\, dt$. Show that the arc length of the curve $y = f(x)$ between $x = a$ and $x = b$ is equal to $g(b) - g(a)$.

10. Find the arc length of the curve $y = \frac{1}{2}(e^x + e^{-x})$ on the interval between the points $(0, 1)$ and $(1, (e + 1/e)/2)$.

11. Find the arc length of the curve $y = \frac{1}{2}x^2 - \frac{1}{4}\ln(x)$ between $x = 1$ and $x = 2$.

12. Find the area of the region in the plane, which (in polar coordinates) is bounded by the lines $\theta = 0$, $\theta = \pi$ and by the portion of the logarithmic spiral $r = e^\theta$ for $0 \leq \theta \leq \pi$.

13. Find the center of mass of a hemisphere of radius R whose base lies in the (x, y)-plane. (Hint: take the center of the hemisphere to lie at the origin. By symmetry you may assume that the center of mass lies on the positive z-axis.)

14. Let C_1 and C_2 be two long cylinders of radius one in space, so that the axis of symmetry of C_1 is the x-axis and the axis of symmetry of C_2 is the y-axis. Find the volume of the intersection of C_1 and C_2. (Hint: each cross-section perpendicular to the z-axis is a square.)