Mathematics 152.02
Review for Midterm 2

1. Solve the following initial value problem:

$$f'(x) = \frac{x^3 + 4x + 1}{x^2 + 3} \quad \text{where} \quad f(0) = 4.$$

2. Use integration by parts to show that for any constants a and b which are not both zero,

$$\int e^{ax} \cos(bx)\, dx = \frac{1}{a^2 + b^2}\, e^{ax} \left(a \cos(bx) + b \sin(bx)\right) + C.$$

3. Find the derivative $f'(x)$ of the function $f(x) = \displaystyle\int_{\sqrt{x}}^{x^3} \frac{t}{1 + e^t}\, dt$.

4. Find the area enclosed by the curve $y = \sqrt{4 - x^2}$, the x-axis, the y-axis and the line $x = 1$.

5. The acceleration of a particle moving in a long thin tube satisfies the equation

$$a(t) = (1 + t)^2$$

for time $t \geq 0$. Find a formula for the position of the particle if it starts at time $t = 0$ at rest at distance 0 from the center of the tube.

6. Define a function by the equation $l(x) = \displaystyle\int_1^x \frac{1}{t}\, dt$. Just using general properties of integrals (i.e., without assuming any properties of $\ln(x)$) show that $l(ab) = l(a) + l(b)$. (Hint: use additivity of the integral and a substitution.)

7. Does the improper integral $\displaystyle\int_1^\infty t^2 e^{-t}\, dt$ converge or diverge? If it converges, find its value.

8. The function $f(x)$ is defined by $f(x) = \displaystyle\int_0^x \sin(1 - t^2)\, dt$. Are the following statements true or false?
   (a) $f(0.5)$ is positive
   (b) $f(x)$ has a maximum at $x = 1$
   (c) $f(x)$ is decreasing near $x = 2$
   (d) $f(x)$ is concave down near $x = 3$