1. Solve the following initial value problem:

\[ f'(x) = \frac{x^3 + 4x + 1}{x^2 + 3} \quad \text{where} \quad f(0) = 4. \]

2. Use integration by parts to show that for any constants \(a\) and \(b\) which are not both zero,

\[ \int e^{ax} \cos(bx) \, dx = \frac{1}{a^2 + b^2} e^{ax} (a \cos(bx) + b \sin(bx)) + C. \]

3. Find the derivative \(f'(x)\) of the function \(f(x) = \int_{\sqrt{2}}^{x^3} \frac{t}{1 + e^t} \, dt\).

4. Find the area enclosed by the curve \(y = \sqrt{4 - x^2}\), the \(x\)-axis, the \(y\)-axis and the line \(x = 1\).

5. The acceleration of a particle moving in a long thin tube satisfies the equation

\[ a(t) = (1 + t)^2 \]

for time \(t \geq 0\). Find a formula for the position of the particle if it starts at time \(t = 0\) at rest at distance 0 from the center of the tube.

6. Define a function by the equation \(l(x) = \int_{1}^{x} \frac{1}{t} \, dt\). Just using general properties of integrals (i.e., without assuming any properties of \(\ln(x)\)) show that \(l(ab) = l(a) + l(b)\). (Hint: use additivity of the integral and a substitution.)

7. Does the improper integral \(\int_{1}^{\infty} t^2 e^{-t} \, dt\) converge or diverge? If it converges, find its value.

8. The function \(f(x)\) is defined by \(f(x) = \int_{0}^{x} \sin(1 - t^2) \, dt\). Are the following statements true or false?
   (a) \(f(0.5)\) is positive
   (b) \(f(x)\) has a maximum at \(x = 1\)
   (c) \(f(x)\) is decreasing near \(x = 2\)
   (d) \(f(x)\) is concave down near \(x = 3\)