Practice Final - Math 1534

1. *(5 points)* Decide if the following statements are TRUE or FALSE and circle your answer. **You do NOT need to justify your answers.**

   T **F** (a) If $\mathbf{u}$ and $\mathbf{v}$ are vectors in $\mathbf{R}^3$ then $(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{u} = 0$.

   T **F** (b) If $\mathbf{r}(t) = \langle r_1(t), r_2(t), r_3(t) \rangle$ is a vector-valued function and $|\mathbf{r}(t)| = 4$ for all $t \in \mathbf{R}$ then $\mathbf{r} \cdot \mathbf{r}' = 0$ for all t.

   T **F** (c) Let $\mathbf{u}$ and $\mathbf{v}$ be a vectors in $\mathbf{R}^3$. Then $\mathbf{u} \cdot (\mathbf{u} - \mathbf{v})$ is a vector in $\mathbf{R}^3$.

   T **F** (d) Let $f = f(x, y)$ be a function of two variables which is continuous at (a, b) and $g = g(t)$ be a function which is continuous at $f(a, b)$. Let
   $$h(t) = f(g(t), g(t)).$$
   Then h is continuous at $f(a, b)$.

   T **F** (e) Let
   $$\sum_{k=0}^{\infty} a_k (x-3)^k$$
   be a power series which converges absolutely at $x = 0$. Then the power series converges at $x = 6$.

2. *(5 points)* Give examples of the following. Be as explicit as possible. **You do NOT need to justify your answers.**

   (a) Give an example of a continuous vector-valued function $\mathbf{r}(t)$ which is *not* differentiable at $t = 2$.

   (b) Give an example of a vector-valued function which has constant curvature $\kappa \neq 0$.

   (c) Give equations for two different planes in $\mathbf{R}^3$ which are parallel to the plane $z = 2x - 5y$.

   (d) Give an example of a vector $\mathbf{u} \in \mathbf{R}^2$ for which $\mathbf{u} \cdot \left\langle \cos \frac{\pi}{5}, \sin \frac{\pi}{5} \right\rangle = 0$

   (e) Give an example of a positive convergent series for which the ratio test is inconclusive.

3. *(5 points)* Decide whether the series
   $$\sum_{k=1}^{\infty} \ln\left(1 + \frac{1}{k^2}\right)$$
   converges absolutely, converges conditionally or diverges.

4. *(3 points)* Give a function f which satisfies
   $$\begin{aligned} f(2) &= 2 \\ f'(2) &= 4 \\ f''(2) &= -3 \\ f'''(2) &= 10. \end{aligned}$$

5. *(4 points)* Let $\mathbf{u} = \langle 3, 2, 2 \rangle$ and $\mathbf{v} = \langle 1, -4, 6 \rangle$.

   (a) Compute $2\mathbf{u} - \mathbf{v}$.

   (b) Compute $\mathbf{u} \cdot \mathbf{v}$.

   (c) Compute $\mathbf{u} \times \mathbf{v}$.

   (d) Compute the angle between $\mathbf{u}$ and $\mathbf{v}$.

   (e) Compute $\text{proj}_{\mathbf{u}} \mathbf{v}$

6. *(5 points)* A ball is thrown from an intial height of $20\,\text{m}$ above with an initial speed of $10\,\text{m/s}$. and initial angle of $\frac{\pi}{3}$ radians with the ground.
   (a) Give the position vector of the ball $\mathbf{r}(t)$ at time t.
   (b) Set up but do not evaluate an integral which gives the length of the path traveled by the ball. This is **not** the distance traveled by the ball along the ground, but the length of the path that the ball traces in the air.

7. *(5 points)* A weight of $5\,\text{N}$ is tied to two strings, both fastened to the ceiling. The first string is tied to the weight at position $\langle 0, 0\rangle$ and is fastened to the ceiling at position $\langle 3, 2\rangle$ The second string is tied to the weight at position $\langle 0, 0\rangle$ and is fastened to the ceiling at position $\langle -1, 2\rangle$. Compute the force vectors which give the forces that the strings exhert on the weight.

8. *(5 points)* Compute the maximum and minimum curvature for the ellipse

$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$

9. *(5 points)* Give an equation of the plane containing the line $\mathbf{r}(t) = \langle 3 + t, 2, 1 - t\rangle$ and the point $\langle 2, -3, 1\rangle$.

10. *(5 points)* Compute the normal and tangential components of acceleration (a_N and a_T) for a particle whose position at time t is given by the vector-valued function

$$\mathbf{r}(t) = \langle t^2, t + 2\rangle.$$

11. *(3 points)* Let $f(x, y) = y^2 e^{xy} - x^3 y$
    (a) Compute the partial derivative f_x.
    (b) Compute $\frac{\partial^2 f}{\partial y \partial x}$.
    (c) Compute $\frac{\partial^2 f}{\partial x \partial y}$.

12. *(5 points)* Give a Taylor polynomial for $f(x) = \cos x$ which estimates $\cos x$ to within $\frac{1}{100}$ for all $x \in [5\pi, 6\pi]$.