Practice Midterm - Math 1534

1. *(5 points)* Decide if the following statements are TRUE or FALSE and circle your answer. **You do NOT need to justify your answers.**

   T **F** (a) If $\sum_{k=0}^{\infty} a_k$ is a convergent series then $\sum_{k=0}^{\infty} a_{2k}$ is a convergent series.

   T **F** (b) If the power series $\sum_{n=0}^{\infty} c_n(x-2)^n$ converges at $x = 4$ then it must converge at $x = -1$.

   T **F** (c) If the radius of convergence of the power series $\sum_{n=0}^{\infty} b_n x^n$ is 0 then the series $\sum_{n=0}^{\infty} b_n$ diverges.

   T **F** (d) For any $r \in \mathbf{R}$ and $\theta \in \mathbf{R}$ the points (r, θ) and $(-r, \theta + \pi)$ represent the same point in polar coordinates.

   T **F** (e) If the Taylor series for f at 0 is

   $$f(x) = \sum_{k=0}^{\infty} a_n x^n$$

   and has radius of convergence 1 then $f^{(4)}(0) = \frac{a_4}{4!}$.

2. *(5 points)* Give examples of the following. Be as explicit as possible. **You do NOT need to justify your answers.**

   (a) Give two **different** ways to represent the point $(x, y) = (-1, -1)$ in polar coordinates.

   (b) Give an example of a power series centered at 3 which has radius of convergence 0.

   (c) Give an example of a p-series that diverges.

   (d) Give two **different** parametrizations of the line $y = 3x + 2$.

   (e) Give an example of a sequence that grows faster than $\{c^n\}_{n=1}^{\infty}$ for any $c > 0$.

3. *(15 points)* Show that the following series converge absolutely, converge conditionally or diverge:

   (a) $\sum_{k=2}^{\infty} \frac{2}{k(\ln k)^4}$

   (b) $\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$

   (c) $\sum_{k=1}^{\infty} \sin \frac{1}{k} \sec^2 \frac{1}{k}$

4. *(10 points)* For the following power series compute the radius of convergence:

   (a) $\sum_{k=1}^{\infty} k^2 x^k$

   (b) $\sum_{n=2}^{\infty} \frac{n x^n}{\ln n}$

5. *(5 points)* Give the interval of convergence of the power series

   $$\sum_{k=1}^{\infty} \frac{(-1)^k (x+2)^k}{k^2}.$$

6. *(5 points)* Give a power series with center 0 for a solution to the differential equation $y' = y + 1$ satisfying the initial condition $y(0) = 1$.

7. Find the equation for the tangent line to the parametric curve

   $$x = \sin t, \quad y = e^t$$

   at $t = 0$.

8. Consider the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

(a) Give the formula for this ellipse in polar coordinates

(b) Derive the area for this ellipse using the polar formula from part (a)

9. Give a partial sum which estimates the series

$$\sum_{k=1}^{\infty} \frac{(-1)^k}{\sqrt[3]{k}}$$

to within $\frac{1}{1000}$. Be sure to justify your answer.

10. Give a Taylor polynomial centered at 0 which estimates

$$f(x) = \ln(1 - x)$$

to within $\frac{1}{10}$ for all $x \in [0, \frac{1}{2}]$ Be sure to justify your answer.