Math 161.01 - Accelerated Calculus I
108 min December 7, 2010

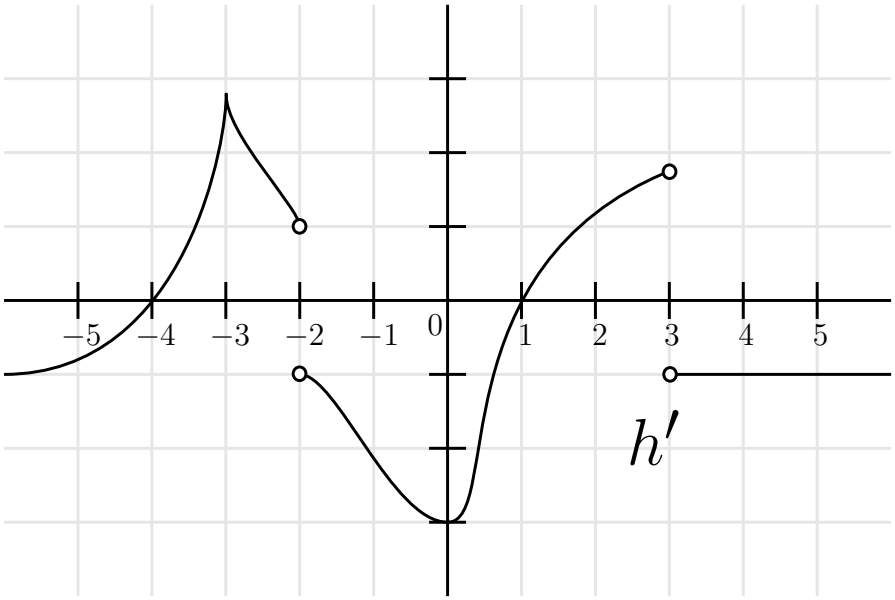
Practice Final

- Calculators are allowed as long as they have no symbolic integration capbility (TI-84 and comparable are ok)
- Remember to **CIRCLE YOUR FINAL ANSWER.**

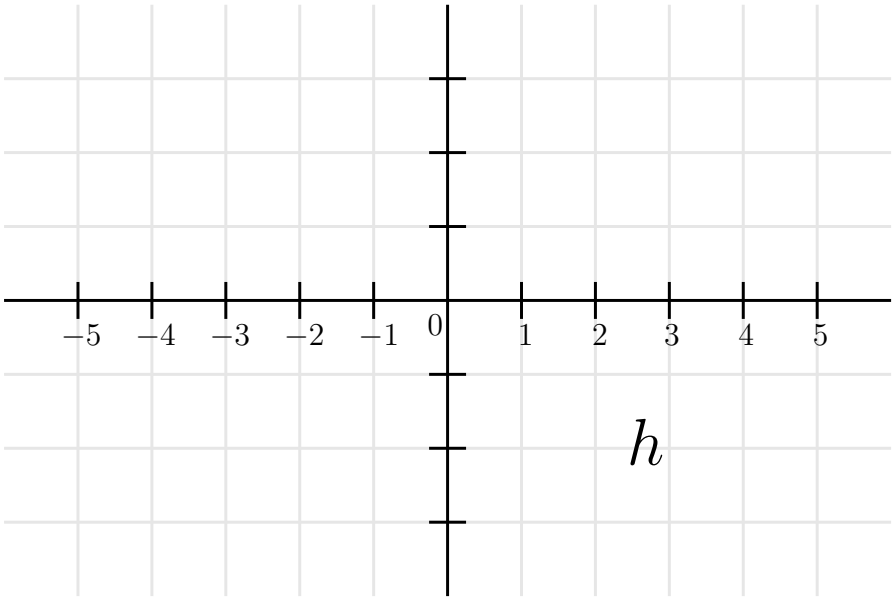
Useful facts:

- Volume of a sphere $V = \frac{4}{3}\pi r^3$
- Volume of a cone $V = \frac{1}{3}\pi r^2 h$
- $F = ma$ (force is mass times acceleration)
- $m = Vd$ (mass is volume times density)
- acceleration of gravity is $-9.8\,\mathrm{m/s^2}$
- $\sin A \sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$
- $\sin A \cos B = \frac{1}{2}[\sin(A-B) + \sin(A+B)]$
- $\cos A \cos B = \frac{1}{2}[\cos(A-B) + \cos(A+B)]$
- $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$
- $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$

1. *(10 points)* The **derivative** of a **continuous** function h is pictured below.



Sketch a **continuous** function h whose derivative could be the given graph for h'.



2. Evaluate the following limits using any technique you like.

   (a) $\lim_{x\to\infty} \sqrt{x+2} - \sqrt{x}$

   (b) $\lim_{x\to\infty} \dfrac{(\ln x)^2}{\ln(\ln x)}$

   (c) $\lim_{x\to\infty} \left(1 + e^{-x}\right)^{\frac{1}{x}}$

3. Evaluate the following derivatives.

   (a) $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(e^x + x^e + \arctan(3 - 8\pi^4)\right)$

   (b) $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(e^x \sin(x^2)\right)$

   (c) $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\dfrac{x}{x+1}\right)$

   (d) $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\dfrac{x^x}{(x+1)^3 \sin x}\right)$

4. Evaluate the following indefinite integrals.

   (a) $\displaystyle\int x \arctan x \,\mathrm{d}x$

   (b) $\displaystyle\int \sin^{17} x \cos^3 x \,\mathrm{d}x$

   (c) $\displaystyle\int \frac{\sec^3 x}{\tan x} \,\mathrm{d}x$

   (d) $\displaystyle\int \sin^4 x \,\mathrm{d}x$

   (e) $\displaystyle\int \cos 2x \cos 9x \,\mathrm{d}x$

   (f) $\displaystyle\int \frac{2x}{\sqrt{x^2 - 9}} \,\mathrm{d}x$

   (g) $\displaystyle\int \frac{x^3}{\sqrt{4 - x^2}} \,\mathrm{d}x$

   (h) $\displaystyle\int \frac{-2x^3 - 6x^2 - 2x + 10}{x^3 + 4x^2 + 5x} \,\mathrm{d}x$

5. Evaluate the following definite integrals

   (a) $\displaystyle\int_1^2 xe^x \,\mathrm{d}x$

(b) $\displaystyle\int_0^{\frac{1}{2}} \sqrt{1-x^2}\,\mathrm{d}x$

6. Let

$$F(x) = \int_{e^x}^{0} \arctan(t+t^2)\,\mathrm{d}t.$$

Find $F'(x)$.

7. *(10 points)* Consider the bounded region between the curves $y^2 = x$ and $x^2 = y$.

   (a) Find the area of the region.

   (b) Using washers give an integral representing the volume when this region is rotated about the line $y = 4$.

   (c) Using cylindrical shells give an integral representing the volume when this region is rotated about the line $x = -2$.

8. How much work is needed to retract a dangling $500\,\mathrm{m}$ chain with a density of $200\,\mathrm{kg/m}$?

9. Find the maximum volume of a cylinder contained in a sphere of radius 1.

10. Water is draining from a spherical tank with radius r (in meters)

   (a) If the water level is at a height of h meters above the center of the tank express the volume of water in the tank **as an integral** involving h.

   (b) If water is draining from the tank at a rate of $2\,\mathrm{m}^3/\mathrm{s}$ then at what rate is the water level changing when the water is $3\,\mathrm{m}$ above the center of the tank?

11. Find the length of the curve $y = \sqrt{1-x^2}$ from $x = 0$ to $x = 1$ using the arc length formula.

12. Consider the curve $y = \sqrt{1-x^2}$ from $x = 0$ to $x = \frac{1}{2}$

   (a) Give an integral but **do not evaluate** for the surface area when the curve is rotated about the line $x = 2$

   (b) Give an integral but **do not evaluate** for the surface area when the curve is rotated about the line $y = 2$