Practice Midterm 1

1. **True or False** (you do not need to justify your answers)
   (a) _____ If f and g are differentiable at a then $f \circ g$ is differentiable at a.
   (b) _____ $\lim_{x\to\infty} \arctan x = 1$.
   (c) _____ If $f'(c) = 0$ and $g'(c) = 0$ then $(f \cdot g)'(c) = 0$.
   (d) _____ If $f(0) = 10$, $f(7) = 4$, and f is continuous on the closed interval $[-1, 8]$ the there must be some number $c \in (-1, 8)$ such that $f(c) = 9$.
   (e) _____ If f' has a vertical asymptote at c then f must have a vertical asymptote

2. Let
$$f(x) = \frac{1}{2x}.$$
   (a) What is $f'(x)$?
   (b) Using **only the definition of the derivative** compute $f'(x)$.

3. Find $\frac{dy}{dx}$ for the following functions
   (a) $y = (10 - e)\sqrt[5]{x^7} - \dfrac{4}{\sqrt[3]{\pi + 4}} + e^2$
   (b) $y = (2x)^{3x}$
   (c) $y = 20 \arcsin\left(\dfrac{2 - 4x}{x^3 + 1}\right)$

4. Show that
$$\lim_{x\to -2} -\tfrac{1}{4}x^3 + 1 = 2$$
   using **only the limit theorems**.

5. Evaluate the following limits using any technique you like.
   (a) $\lim_{x\to 0} \dfrac{3x^2 + 7x}{2x + 5}$
   (b) $\lim_{x\to\infty} \dfrac{\sqrt{x^2 + \sqrt[3]{2x^6 + 1}}}{x + 5}$

6. Consider the curve $y^3 = x^4 y - 3x + 9$
   (a) Find $\frac{dy}{dx}$ in terms of x and y
   (b) Find $\frac{d^2y}{dx^2}$ solely in terms of x and y
   (c) Find the equation for the tangent line to the curve through the point $(1, 2)$

7. A particle's position at time t is given by the equation
$$s(t) = \cos(p\pi t).$$
   (a) Find the average velocity of the particle on the time interval from $t = 0$ to $t = \frac{2}{p}$
   (b) Give an equation for the acceleration of the particle at time t.

8. Simplify the following expressions
   (a) $\sec(\tan^{-1} x)$
   (b) $\sin(\cos^{-1} x)$