Practice Midterm 1

1. **True or False** (you do not need to justify your answers)

(a) _____ If \( f \) and \( g \) are differentiable at \( a \) then \( f \circ g \) is differentiable at \( a \).

(b) _____ \( \lim_{x \to \infty} \arctan x = 1 \).

(c) _____ If \( f'(c) = 0 \) and \( g'(c) = 0 \) then \( (f \cdot g)'(c) = 0 \).

(d) _____ If \( f(0) = 10 \), \( f(7) = 4 \), and \( f \) is continuous on the closed interval \([-1, 8]\) the there must be some number \( c \in (-1, 8) \) such that \( f(c) = 9 \).

(e) _____ If \( f' \) has a vertical asymptote at \( c \) then \( f \) must have a vertical asymptote.

2. Let 

\[ f(x) = \frac{1}{2x} \]

(a) What is \( f'(x) \)?

(b) Using only the definition of the derivative compute \( f'(x) \).

3. Find \( \frac{dy}{dx} \) for the following functions

(a) \( y = (10 - e)\sqrt{x} - \frac{4}{\sqrt{x} + 4} + e^2 \)

(b) \( y = (2x)^3x \)

(c) \( y = 20 \arcsin \left( \frac{2 - 4x}{x^3 + 1} \right) \)

4. Show that

\[ \lim_{x \to -2} -\frac{1}{4}x^3 + 1 = 2 \]

using only the limit theorems.

5. Evaluate the following limits using any technique you like.

(a) \( \lim_{x \to 0} \frac{3x^2 + 7x}{2x + 5} \)

(b) \( \lim_{x \to \infty} \frac{\sqrt{x^2 + 2x^4 + 1}}{x + 5} \)

6. Consider the curve \( y^3 = x^4y - 3x + 9 \)

(a) Find \( \frac{dy}{dx} \) in terms of \( x \) and \( y \)

(b) Find \( \frac{d^2y}{dx^2} \) solely in terms of \( x \) and \( y \)

(c) Find the equation for the tangent line to the curve through the point \( (1, 2) \)

7. A particle’s position at time \( t \) is given by the equation

\[ s(t) = \cos(pt) \]

(a) Find the average velocity of the particle on the time interval from \( t = 0 \) to \( t = \frac{2}{p} \)

(b) Give an equation for the acceleration of the particle at time \( t \).

8. Simplify the following expressions

(a) \( \sec(\tan^{-1} x) \)

(b) \( \sin(\cos^{-1} x) \)