Practice Midterm 1 – Math 2153

1. Decide if the following statements are TRUE or FALSE and circle your answer. **You do NOT need to justify your answers.**
   (a) (1 point) If $\mathbf{u}$ and $\mathbf{v}$ are vectors in $\mathbf{R}^3$ then $(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{u} = 0$.
   (b) (1 point) If $\mathbf{r}(t) = \langle r_1(t), r_2(t), r_3(t) \rangle$ is a vector-valued function and $|\mathbf{r}(t)| = 4$ for all $t \in \mathbf{R}$ then $\mathbf{r} \cdot \mathbf{r}' = 0$ for all t.
   (c) (1 point) Let $\mathbf{u}$ and $\mathbf{v}$ be a vectors in $\mathbf{R}^3$. Then $\mathbf{u} \cdot (\mathbf{u} - \mathbf{v})$ is a vector in $\mathbf{R}^3$.
   (d) (1 point) Let $f = f(x, y)$ be a function of two variables which is continuous at (a, b) and $g = g(t)$ be a function which is continuous at $f(a, b)$. Let
   $$h(t) = f(g(t), g(t)).$$
   Then h is continuous at $f(a, b)$.

2. Give examples of the following. Be as explicit as possible. **You do NOT need to justify your answers.**
   (a) (2 points) Give an example of a continuous vector-valued function $\mathbf{r}(t)$ which is *not* differentiable at $t = 2$.
   (b) (2 points) Give an example of a vector-valued function which has constant curvature $\kappa \neq 0$.
   (c) (2 points) Give equations for two different planes in $\mathbf{R}^3$ which are parallel to the plane $z = 2x - 5y$.
   (d) (2 points) Give an example of a vector $\mathbf{u} \in \mathbf{R}^2$ for which $\mathbf{u} \cdot \left\langle \cos\frac{\pi}{5}, \sin\frac{\pi}{5} \right\rangle = 0$

3. Let $\mathbf{u} = \langle 3, 2, 2 \rangle$ and $\mathbf{v} = \langle 1, -4, 6 \rangle$.
   (a) (2 points) Compute $2\mathbf{u} - \mathbf{v}$.
   (b) (2 points) Compute $\mathbf{u} \cdot \mathbf{v}$.
   (c) (2 points) Compute $\mathbf{u} \times \mathbf{v}$.
   (d) (2 points) Compute the angle between $\mathbf{u}$ and $\mathbf{v}$.
   (e) (2 points) Compute $\mathrm{proj}_{\mathbf{u}}\mathbf{v}$

4. A ball is thrown from an intial height of $20\,\mathrm{m}$ above with an initial speed of $10\,\mathrm{m/s}$. and initial angle of $\frac{\pi}{3}$ radians with the ground.
   (a) (3 points) Give the position vector of the ball $\mathbf{r}(t)$ at time t.
   (b) (2 points) Set up but do not evaluate an integral which gives the length of the path traveled by the ball. This is **not** the distance traveled by the ball along the ground, but the length of the path that the ball traces in the air.

5. (5 points) A weight of $5\,\mathrm{N}$ is tied to two strings, both fastened to the ceiling. The first string is tied to the weight at position $\langle 0, 0 \rangle$ and is fastened to the ceiling at position $\langle 3, 2 \rangle$ The second string is tied to the weight at position $\langle 0, 0 \rangle$ and is fastened to the ceiling at position $\langle -1, 2 \rangle$. Compute the force vectors which give the forces that the strings exhert on the weight.

6. (5 points) Compute the maximum and minimum curvature for the ellipse
   $$\frac{x^2}{4} + \frac{y^2}{9} = 1.$$

7. (5 points) Give an equation for the plane containing the line $\mathbf{r}(t) = \langle 3 + t, 2, 1 - t \rangle$ and the point $\langle 2, -3, 1 \rangle$.

8. (5 points) Compute the normal and tangential components of acceleration (a_N and a_T) for a particle whose position at time t is given by the vector-valued function
   $$\mathbf{r}(t) = \langle t^2, t + 2 \rangle.$$