Practice Midterm 2 – Math 2153

1. Decide if the following statements are TRUE or FALSE and circle your answer. **You do NOT need to justify your answers.**

   (a) (1 point) If both partial derivatives f_x and f_y exist at (a, b) then f is differentiable at (a, b).

   (b) (1 point) If f has a local maximum at the point (a, b, c) then $\nabla f = \mathbf{0}$.

   (c) (1 point) If f has a saddle point at (a, b) then f cannot have a local minimum at (a, b).

   (d) (1 point) If f is differentiable at (a, b, c) then magnitude of the gradient vector $\nabla f(a, b, c)$ is the maximal directional derivative
   $$D_{\mathbf{u}}(a, b, c)$$
   where $\mathbf{u}$ ranges over all unit vectors in $\mathbf{R}^3$.

2. Give examples of the following. Be as explicit as possible. **You do NOT need to justify your answers.**

   (a) (2 points) Give an example of a function $f(x, y)$ continuous on $\mathbf{R}^2$ such that there are infinitely many points $(a, b) \in \mathbf{R}^2$ such that f has a local maximum at (a, b).

   (b) (2 points) Give an example of a function $F(x, y, z)$ for which the graph of $z = \sin(xy)$ is a level surface.

   (c) (2 points) Give an example of a function $f(x, y)$ with domain $\mathbf{R}^2$ for which $f_x(0, 0)$ exists but $f_y(0, 0)$ does not exist.

   (d) (2 points) Sketch level curves for a function $f(x, y)$ with four local maximuma and no local minima. Make sure to include enough level curves to illustrate these properties.

3. Compute the following:

   (a) (2 points) $\dfrac{\partial}{\partial y}\left(\dfrac{\ln(xy)}{x^y}\right)$

   (b) (2 points) $D_{\mathbf{u}}(3x^3y - y)$ where $\mathbf{u} = \left\langle \frac{-2}{\sqrt{5}}, \frac{-1}{\sqrt{5}} \right\rangle$.

   (c) (2 points) Find the unit vector $\mathbf{u} = \langle u_1, u_2, u_3 \rangle$ pointing in the direction of maximum increase for the function $f(x, y, z) = xyz^2$ at the point $(1, 1, 1)$.

   (d) (2 points) $\displaystyle\int_1^2 \int_0^\pi \sin(x + y)\, dx\, dy$.

   (e) (2 points) $\displaystyle\int_1^2 \int_0^2 \int_0^1 \frac{xz}{y}\, dz\, dx\, dy$.

   (f) (2 points) Compute $\frac{\partial z}{\partial x}$ in terms of x, y and z if z satisfies the implicit equation
   $$xy + yz + xz = 7$$

4. Change the order of integration for the following double integrals. You may have to express the new integral as a sum of double integrals. **You do not need to evaluate the integrals.**

   (a) (2 points) $\displaystyle\int_1^2 \int_{x-1}^{\ln x} xy^2\, dy\, dx$.

   (b) (2 points) $\displaystyle\int_0^{\pi/4} \int_{\sin y}^{\cos y} xy^2\, dx\, dy$.

5. (5 points) Give a triple integral which computes the volume of the bounded region in $\mathbf{R}^3$ enclosed by the surfaces
   $$x^2 + y^2 - z^2 = 9, \quad z = 4, \quad z = -4$$

6. (5 points) Compute the maximum and minimum values for the function $f(x, y) = x - y^2 + xy$ on the region in the plane bounded by the ellipse $y^2 + 9x^2 = 9$.

7. (5 points) Use the method of Lagrange multipliers to find the maximum volume for a lidless can with surface area 1.

8. (5 points) Estimate the change in $z = xy^2 - x^2 + y$ when (x, y) changes from $(1, 2)$ to $(1.1, 1.9)$.