Calculus III – Recitation 1

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12 Vectors and vector-valued functions

1. Let $P = (-2, 3)$ and $Q = (-4, 10)$.
   (a) Compute $|\overrightarrow{PQ}|$.
   (b) Compute $|\overrightarrow{QP}|$.
   (c) Is it true that for any points R and S in the plane
   $$|\overrightarrow{RS}| = |\overrightarrow{SR}|?$$
   Prove this or give a counterexample.
2. A **unit vector** is a vector with magnitude 1.
   (a) Give a unit vector which makes an angle of $\pi/5$ with the x-axis.
   (b) Compute $|\langle 1, 2\rangle|$.
   (c) Compute $|5\langle 1, 2\rangle|$.
   (d) Give a formula for $|c\mathbf{v}|$ if c is a scalar and $\mathbf{v}$ is a vector.
   (e) Give a unit vector which is parallel to the vector $\langle 1, 2\rangle$.
   (f) Give a formula for a unit vector which is parallel to the vector $\mathbf{v}$.
3. Let $\mathbf{i} = \langle 1, 0\rangle$ and $\mathbf{j} = \langle 0, 1\rangle$ be the **coordinate unit vectors** and let $\mathbf{u} = \langle 1, 1\rangle$.
   (a) Write the vector $\langle -3, 4\rangle$ in the form $a\mathbf{i} + b\mathbf{j}$ where a and b are scalars.
   (b) Can every vector $\mathbf{v} = \langle v_1, v_2\rangle$ be written in the form $a\mathbf{i} + b\mathbf{j}$ where a and b are scalars?
   (c) Describe the set of vectors which can be written in the form $a\mathbf{i}$ where a is a scalar.
   (d) Write the vector $\langle -3, 4\rangle$ in the form $a\mathbf{i} + b\mathbf{j} + c\mathbf{u}$ where a, b and c are scalars in two different ways.
   (e) $\langle -3, 4\rangle$ in the form $a\mathbf{i} + c\mathbf{u}$ where a and c are scalars.
4. Triangle inequality
   (a) Compute $|\mathbf{i} + \mathbf{j}|$.
   (b) Compute $|\mathbf{i}| + |\mathbf{j}|$.
   (c) Is it true that $|\mathbf{u} + \mathbf{v}| = |\mathbf{u}| + |\mathbf{v}|$ for all vectors $\mathbf{u}$ and $\mathbf{v}$?
   (d) Give a nonzero vector $\mathbf{v}$ such that
   $$|\langle 0, 2\rangle + \mathbf{v}| = |\langle 0, 2\rangle| + |\mathbf{v}|.$$

5. Solve for the vectors $\mathbf{u}$ and $\mathbf{v}$

$$\begin{aligned} \mathbf{u} + \mathbf{v} &= \langle 3, 0 \rangle \\ \mathbf{u} - \mathbf{v} &= \langle 1, 2 \rangle \end{aligned}$$

6. Consider the 12 vectors that have their tails at the center of a circular clock and their heads at the numbers on the edge of the clock.
   (a) What is the sum of these 12 vectors?
   (b) If the 12:00 vector is removed what is the sum of the 11 remaining vectors?
   (c) By removing one or more of these 12 vectors explain how to make the remaining vectors as large as possible in magnitude.
   (d) Consider the 11 vectors that originate at the number 12 and point to the other 11 numbers. What is the sum of the vectors?