

1. Compute the curvature $\kappa(t)$ of the vector-valued function

$$\mathbf{r}(t) = t \mathbf{i} + \sin t \mathbf{j}.$$

2. Compute the torsion $\tau(t)$ of the vector-valued function

$$\mathbf{r}(t) = \arctan t \mathbf{i} + \frac{\mathbf{j}}{1+t^2}.$$

3. Find the curvature at θ of the polar curve given by the equation $r = \theta$.

4. Find the tangential and normal components of acceleration for the trajectory

$$\mathbf{r}(t) = t^3 \mathbf{i} + t^2 \mathbf{j}.$$

5. Let $s(t)$ be the arc length function for the vector-valued function

$$\mathbf{r}(t) = \ln t \mathbf{i} + t^{-1} \mathbf{j} + t \mathbf{k}.$$

Compute $\frac{ds}{dt}$ at $t = 2$.

6. Decide if the points $A = (1, 0, 2)$, $B = (-1, 1, 1)$ and $C = (5, -2, 6)$ are colinear.

7. Decide if the points $A = (-5, 1, 1)$, $B = (-7, -1, 0)$, $C = (4, 2, 2)$ and $C = (-1, 0, 2)$ are coplanar.

8. Find the point in the plane

$$2x - 4y + z = 3$$

which is closest to the point $A = (1, 3, 5)$.

9. A light ray travels along the line

$$\mathbf{r}(t) = (3 - t) \mathbf{i} + t \mathbf{j} + (2 + t) \mathbf{k}$$

until it is reflected by the plane

$$x - y + z = 3$$

- (a) At what value of t does the light ray hit the plane? What is the point of intersection.
- (b) If the angle of incidence equals the angle of reflection, what is the new direction vector of the light ray after reflection by the plane? Give the line along which the reflected light ray travels. (*Hint: you may need to use orthogonal projection*)
- (c) Give a general formula for the reflection of the vector \mathbf{v} in the plane through the origin $\mathbf{0}$ with normal vector \mathbf{n} .