Math 2153 – Recitation 9 October 29, 2015

1. Evaluate the following double integrals

   (a) $\iint_R \frac{y^2}{x^2}\, dA$ where $R = \{(x,y) \mid 1 \le x \le 3 \text{ and } 2 \le y \le 4\}$.

   (b) $\iint_R x^y\, dA$ where $R = \{(x,y) \mid 1 \le x \le 3 \text{ and } 2 \le y \le 4\}$.

2. Let D be the bounded region in $\mathbf{R}^3$ enclosed by the planes

$$x = y, \quad z = y - 5, \quad z = 0 \quad \text{and} \quad x = 0.$$

   (a) The region enclosed is a tetrahedron. Find its four corners.

   (b) Give a double integral which represents the volume of D.

   (c) Give a triple integral which represents the volume of D.

3. Reverse the order of integration for the following double integrals. You may need to split the integral into a sum of multiple double integrals.

   (a) $\int_0^{\frac{\pi}{3}} \int_{\tan x}^{\sqrt{3}} f(x,y)\, dy\, dx$

   (b) $\int_1^6 \int_{\ln y}^{y} f(x,y)\, dx\, dy$

   (c) $\int_0^4 \int_{x^2-4x}^{4x-x^2} f(x,y)\, dy\, dx$

4. Find the average of the squares of the distances from the points on the annulus

$$R = \{(x,y) \mid 4 \le x^2 + y^2 \le 16\}$$

   to the origin.

5. Find the average of the distances from the points on the annulus

$$R = \{(x,y) \mid 4 \le x^2 + y^2 \le 16\}$$

   to the origin.

6. Evaluate $\iint_R e^{x^2+y^2}\, dA$ where $R = \{(x,y) \mid x^2 + y^2 \le 9\}$.

7. Let $0 \le a < b$ and $f(r,\theta) = \frac{r-a}{b-a}$. If R is the polar rectangle

$$R = \{(r,\theta) \mid \alpha \le \theta \le \beta \text{ and } a \le r \le b\}$$

   do you expect the average value of f on R to be $\frac{1}{2}$, greater than $\frac{1}{2}$ or less than $\frac{1}{2}$? Compute the average value of f on R.