

1. Find the centroid for the following objects:

(a) Let  $D$  be a solid cone with height  $h$  base radius  $r$  and constant density.

(b) Let  $R$  be a plate in the shape of the bounded region enclosed by the functions  $y = x^2 - 1$  and  $y = \frac{3x^2}{4} - \frac{7}{16}$  with constant density function.

2. Let  $D$  be a bounded object enclosed by the planes

$$x = y, \quad z = y - 5, \quad z = 0 \quad \text{and} \quad x = 0.$$

with density function  $f(x, y, z) = |x + y + z| + 1$ .

(a) Find the total mass of  $D$ .

(b) Find the  $y$ -coordinate of the centroid of  $D$ .

3. The force of gravity that a point mass with mass  $m_1$  exerts on a point mass with mass  $m_2$  at a distance  $r$  has magnitude

$$|\mathbf{F}| = \frac{Gm_1m_2}{r^2}$$

where  $G$  is the universal gravitational constant (and has value approximately  $6.674 \times 10^{-11} \text{Nm}^2/\text{kg}^2$ ) Set up a spherical integral which computes the force of gravity of a spherical planet with constant density  $c$  in  $\text{kg}/\text{m}^3$  and radius  $R$  in  $\text{m}$  on a point mass of mass  $m$  located  $D$  meters from the center of the planet.

4. Evaluate  $\iint_R x^2 - xy + y^2 dA$  where  $R$  is the region bounded by the ellipse  $x^2 - xy + y^2 = 2$ . Use the change of variables  $x = \sqrt{2}u - \sqrt{\frac{2}{3}}v$ ,  $y = \sqrt{2}u + \sqrt{\frac{2}{3}}v$ .