Math 2153 – Recitation 13 December 3, 2015

1. Decide if the vector field $\mathbf{F}$ is conservative on the given domain
   (a) $\mathbf{F}(x, y) = \langle 3x^2y^2, 2x^3y\rangle$ on the domain $\mathbf{R}^2$.
   (b) $\mathbf{F}(x, y) = \left\langle \frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2} \right\rangle$ on the domain $\{(x, y) \,|\, x > 0\}$.
   (c) $\mathbf{F}(x, y) = \left\langle \frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2} \right\rangle$ on the domain $\mathbf{R}^2 - \{(0, 0)\}$. *(Hint: Use the criterion that $\oint_C \mathbf{F} \cdot d\mathbf{r} = 0$ for all closed loops C)*
   (d) $\mathbf{F}(x, y) = \langle xy, xy\rangle$ on the domain $\mathbf{R}^2$.
   (e) $\mathbf{F}(x, y, z) = \langle y^2z^3, 2xyz^3 + 6yz, 3xy^2z^2 + 3y^2\rangle$ on the domain $\mathbf{R}^3$.
   (f) $\mathbf{F}(x, y, z) = \langle x, y, z\rangle$ on the domain $\mathbf{R}^3$.
2. For the conservative vector fields in Problem 1 find a potential function.
3. For what values of the constants a, b, c and d is the vector field

$$\mathbf{F}(x, y) = \langle ax + by, cx + dy\rangle$$

   conservative on $\mathbf{R}^2$?
4. Let $\mathbf{F} = \langle -y, 0\rangle$.
   (a) Evaluate the circulation of the vector field $\mathbf{F}$ along the closed loop C where C is the boundary curve of the triangle with vertices $(-1, 1)$, $(3, 2)$, and $(0, 4)$ oriented counterclockwise.
   (b) Use Green's theorem to give a double integral which computes this circulation.
   (c) Are line integrals in the vector field $\mathbf{F}$ path independant in the domain $\mathbf{R}^2$?
5. Let $\mathbf{F} = \langle f, g\rangle$ be a vector field with continuous first partial derivatives and let

$$\mathbf{G} = \langle -g, f\rangle.$$

   (a) What is the geometric relationship between the vector fields $\mathbf{F}$ and $\mathbf{G}$?
   (b) How are the 2-dimensional curl and divergence for the vector fields $\mathbf{F}$ and $\mathbf{G}$ related?
   (c) If the vector field $\mathbf{F}$ is irrotational what can you conclude about the vector field $\mathbf{G}$? What if $\mathbf{F}$ is source-free?
   (d) Prove the flux version of Green's Theorem for $\mathbf{F}$ using the circulation version of Green's Theorem for $\mathbf{G}$.
6. (a) Must a conservative vector field on a region R in $\mathbf{R}^2$ be irrotational?
   (b) Must a irrotational vector field on a region R in $\mathbf{R}^2$ be conservative?