1. Decide if the vector field \( \mathbf{F} \) is conservative on the given domain
   
   (a) \( \mathbf{F}(x, y) = \langle 3x^2y^2, 2x^3y \rangle \) on the domain \( \mathbb{R}^2 \).
   
   (b) \( \mathbf{F}(x, y) = \left\langle \frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2} \right\rangle \) on the domain \( \{(x, y) | x > 0\} \).
   
   (c) \( \mathbf{F}(x, y) = \left\langle \frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2} \right\rangle \) on the domain \( \mathbb{R}^2 - \{(0,0)\} \). (Hint: Use the criterion that \( \oint_C \mathbf{F} \cdot d\mathbf{r} = 0 \) for all closed loops \( C \)).
   
   (d) \( \mathbf{F}(x, y) = \langle xy, xy \rangle \) on the domain \( \mathbb{R}^2 \).
   
   (e) \( \mathbf{F}(x, y, z) = \langle y^2z^3, 2xyz^3 + 6yz, 3xy^2z^2 + 3y^2 \rangle \) on the domain \( \mathbb{R}^3 \).
   
   (f) \( \mathbf{F}(x, y, z) = \langle x, y, z \rangle \) on the domain \( \mathbb{R}^3 \).

2. For the conservative vector fields in Problem 1 find a potential function.

3. For what values of the constants \( a, b, c \) and \( d \) is the vector field
   \( \mathbf{F}(x, y) = \langle ax + by, cx + dy \rangle \)
   conservative on \( \mathbb{R}^2 \)?

4. Let \( \mathbf{F} = \langle -y, 0 \rangle \).
   
   (a) Evaluate the circulation of the vector field \( \mathbf{F} \) along the closed loop \( C \) where \( C \) is the boundary curve of the triangle with vertices \((-1, 1), (3, 2), \) and \((0, 4)\) oriented counterclockwise.
   
   (b) Use Green’s theorem to give a double integral which computes this circulation.
   
   (c) Are line integrals in the vector field \( \mathbf{F} \) path independent in the domain \( \mathbb{R}^2 \)?

5. Let \( \mathbf{F} = \langle f, g \rangle \) be a vector field with continuous first partial derivatives and let
   \( \mathbf{G} = \langle -g, f \rangle \).
   
   (a) What is the geometric relationship between the vector fields \( \mathbf{F} \) and \( \mathbf{G} \)?
   
   (b) How are the 2-dimensional curl and divergence for the vector fields \( \mathbf{F} \) and \( \mathbf{G} \) related?
   
   (c) If the vector field \( \mathbf{F} \) is irrotational what can you conclude about the vector field \( \mathbf{G} \)? What if \( \mathbf{F} \) is source-free?
   
   (d) Prove the flux version of Green’s Theorem for \( \mathbf{F} \) using the circulation version of Green’s Theorem for \( \mathbf{G} \).

6. (a) Must a conservative vector field on a region \( R \) in \( \mathbb{R}^2 \) be irrotational?
   
   (b) Must a irrotational vector field on a region \( R \) in \( \mathbb{R}^2 \) be conservative?