Practice Midterm 2 – Math 2255

$**$ **You will be allowed to bring a single double-sided 8.5×11 page of notes for this midterm**.

1. Decide if the following statements are TRUE or FALSE. **You do NOT need to justify your answers.**
   (a) (1 point) Any periodic function with period T can be written in the form
   $$f(t) = R\cos\left(\frac{2\pi t}{T} + \delta\right)$$
   for some constants R and δ.

2. Give examples of the following. Be as explicit as possible. **You do NOT need to justify your answers.**
   (a) (2 points) Give an example of a piecewise continuous function on the interval $[0, 5]$ which is not continuous on the interval $[0, 5]$.

3. (5 points) Find the smallest vector space of functions closed under differentiation which contains the function
   $$f(t) = t^2 \sin 2t.$$
   You do NOT need to justify your answer.

4. (5 points) Find the smallest vector space of functions closed under the linear operator $t^2 D^2$ which contains the function
   $$f(t) = t^4$$
   You do NOT need to justify your answer.

5. (5 points) Find the values R and δ for which
   $$\cos t - 2\sin t = R\cos(t - \delta)$$
   for all $t \in \mathbf{R}$.

6. Find general solutions for the following differential equations
   (a) (5 points) $y'' - 4y' + 5y = t^2$.
   (b) (5 points) $y'' - 4y' + 4y = te^{2t}$.
   (c) (5 points) $y'' - 5y' - 6y = 3t - e^{-t}$.
   (d) (5 points) $y'' - 4y' + 13y = e^t \cos t$.

7. Find solutions for the following initial or boundary value problems
   (a) (5 points) $y'' + 3y' = e^{-3t}, \quad y(0) = 0, \quad y'(0) = 1$.
   (b) (5 points) $y'' - y' - 5y = 0, \quad y'(0) = 0, \quad y(1) = e$.
   (c) (5 points) $y'' + 7y = 6, \quad y(0) = 0, \quad y(1) = 3$.

8. Find the general solution to
   $$t^2 y'' - 2y = 3t^2 - 1$$
   given that the general solution to the homogeneous equation $4t^2 y'' - 2y = 0$ is
   $$c_1 t^2 + c_2 t^{-1}.$$

9. (10 points) Using only the definition of the Laplace transform compute the Laplace transform $\mathcal{L}\{t^2\}$.

10. (10 points) When a $10\,\mathrm{kg}$ mass is attached to a spring the spring stretches $0.5\,\mathrm{m}$. Find the natural angular frequency ω_0 of the spring if there is no damping. Express your answer as a function of acceleration g. (Do not substitute $9.8\,\mathrm{m/s^2}$ in for g)

11. (10 points) Find the inverse Laplace transform $\mathcal{L}^{-1}\left\{\frac{1}{(s-2)(s-3)}+\frac{1}{(s-4)^3}\right\}$.

12. (10 points) An external force of the form $F(t) = F_0 \cos \omega t$ is applied to a $2\,\mathrm{kg}$ mass attached to a spring with spring constant $k = 8\,\mathrm{N/m}$ where $F_0 = 5\,\mathrm{N}$. The system has a damping constant of $\gamma = 2\,\mathrm{Ns/m}$. Find the frequency ω at which the system will exhibit resonance. What amplitude do you expect for the motion at that resonant frequency.

13. (10 points) Solve the initial value problem

$$y'' + 4y = g(t)$$

where

$$g(t) = \begin{cases} 0, & 0 \le t < 2 \\ 1, & 2 \le t \end{cases}$$

and

$$y(0) = 0, \qquad y'(0) = 0.$$