

## Practice Final – Math 2255

**\*\* Practice midterms I and II** provide good practice problems for previous material.

**\*\* Bring TWO double-sided  $8.5 \times 11$  sheets of notes to use during the final.**

1. Decide if the following statements are TRUE or FALSE. **You do NOT need to justify your answers.**

(a) (2 points) Let  $L_1$  and  $L_2$  be linear second order linear differential operators. If  $f(t)$  is a solution to the differential equations

$$L_1[y] = 0$$

and  $g(t)$  is a solution to the differential equation

$$L_2[y] = 0$$

then  $f(t) + g(t)$  is a solution to the differential equation

$$(L_1 + L_2)[y] = 0.$$

2. Give examples of the following. Be as explicit as possible. **You do NOT need to justify your answers.**

(a) (2 points) Give an example of a linear differential equation for which  $x_0 = 4$  is a regular singular point.

(b) (2 points) Give an example of a linear differential equation for which  $x_0 = 4$  is an irregular singular point.

3. (10 points) Compute the radius of convergence for the power series

$$\sum_{n=0}^{\infty} \frac{n^2 6^n (x-4)^n}{(n+1)8^{n+2}}$$

4. (10 points) Find the general solution to the differential equation

$$x^2 y'' - 3xy' + 3y = x^4.$$

5. (10 points) Give a positive lower bound with justification for the radius of convergence for any series solution to the differential equation

$$y'' - \frac{y'}{e - e^x} + \frac{y}{4x^2 + 1} = 0$$

centered at  $x = -1$ .

6. (10 points) Find the general series solution for the differential equation

$$y'' + x^3 y = 0$$

centered at  $x = 0$ .

7. (10 points) Find a nonzero series solution to the equation

$$x^2 y'' + xy' + (-1 + x)y = 0$$

centered at  $x_0 = 0$ .

8. (10 points) What is the indicial equation for the differential equation

$$x^2 y'' + \frac{(\cos x - 1)y'}{x} + \frac{(\sin x)y}{3x} = 0$$

at  $x = 0$ .