Practice Midterm 1 – Math 2255

*****Bring a two sided 8.5×11 sheet of notes (definitions, theorems and formulas are ok but no worked examples) to use during Midterm 1.**

1. Decide if the following statements are TRUE or FALSE. **You do NOT need to justify your answers.**
   (a) (1 point) If the functions φ and ψ are solutions to the differential equation
   $$y'' + ay = 0$$
   then $\varphi - 2\psi$ is a solution to the differential equation as well.
   (b) (1 point) If either of the functions p or g is not differentiable at $t = t_0$ then the initial value problem
   $$y' + p(t)y = g(t), \qquad y(t_0) = y_0$$
   cannot have a unique solution on any open interval I containing t_0.
   (c) (1 point) It is possible for $\varphi(t) = e^t$ and $\psi(t) = 1$ to be solutions to the same first order differential equation
   $$y' = f(y, t)$$
   where f and $\frac{\partial f}{\partial y}$ are continuous on the entire (y, t)-plane.
   (d) (1 point) Let p and q be continuous on $\mathbf{R}$. Let y_1 and y_2 be solutions to the differential equation
   $$y'' + p(t)y' + q(t)y = 0.$$
   If the Wronskian determinant $W(y_1, y_2)(t_0)$ is 0 at t_0 then
   $$W(y_1, y_2)(t) = 0$$
   for all $t \in \mathbf{R}$.
2. Give examples of the following. Be as explicit as possible. **You do NOT need to justify your answers.**
   (a) (2 points) Give an example of a first order differential equation which is separable but not linear.
   (b) (2 points) Give an example of a nonhomogeneous 3rd order linear differential equation with **constant** coefficients.
   (c) (2 points) Give an example of a first order autonomous differential equation with exactly two equilibrium solutions.
3. (5 points) Compute the Wronskian for $y_1(t) = t^2$, $y_2(t) = \frac{1}{t}$
4. (5 points) Find the smallest vector space of functions closed under differentiation which contains the function
   $$f(t) = t^2 \sin 2t.$$
5. (5 points) Find the integrating factor for the first order linear equation
   $$ty' - 6y - t^2 + 1 = 0.$$
6. Find general solutions for the following differential equations
   (a) (5 points) $y' = (1 + y^2)e^x$.
   (b) (5 points) $y'' - 4y' + 4y = 0$.
   (c) (5 points) $y'' - 5y' - 6y = 0$.
   (d) (5 points) $y'' - 4y' + 13y = 0$.
   (e) (5 points) $y' - t^3 y + t^3 = 0$.

(f) (5 points) $y'' - 4y' + 5y = t^2$.

(g) (5 points) $y'' - 4y' + 4y = te^{2t}$.

(h) (5 points) $y'' - 5y' - 6y = 3t - e^{-t}$.

(i) (5 points) $y'' - 4y' + 13y = e^t \cos t$.

7. Find solutions for the following initial value problems

   (a) (5 points) $y'' + 3y' = 0, \quad y(0) = 3, \quad y'(0) = 1$.

   (b) (5 points) $y'' + 6y' + 10y = 0, \quad y(0) = 1, \quad y'(0) = 0$.

   (c) (5 points) $ty' + y = 6, \quad y(1) = 3$.

   (d) (5 points) $y'' + 8y' + 20y = 0, \quad y(0) = 6, \quad y'(0) = -1$.

   (e) (5 points) $y'' + 3y' = e^{-3t}, \quad y(0) = 0, \quad y'(0) = 1$.

   (f) (5 points) $y'' - y' - 5y = 0, \quad y(0) = e, \quad y'(0) = 0$.

   (g) (5 points) $y'' + 7y = 6, \quad y(0) = 0, \quad y'(0) = 3$.

8. (10 points) Sketch solutions to the equation

$$y' = \ln |y|.$$

   on the (y, t)-plane. Be sure to include all equilibrium solutions and note whether they are stable, unstable or semistable. Also include a nonequilibrium solution above and below each equilibrium solution.

9. (10 points) t^2 is a solution to the differential equation

$$3t^2 y'' - 2ty' - 2y = 0.$$

   Find the general solution.

10. (10 points) Suppose that the force of drag on a ball of mass m falling through the air were proportional to the velocity v of the ball. Set up a model for the velocity of the ball falling through the air if the acceleration of gravity is g. Find the velocity as a function of time if the initial velocity is 0.

11. (10 points) Show that the functions $y_1(t) = t$ and $y_2(t) = \sin(t)$ cannot both be solutions to a single differential equation of the form

$$y'' + p(t)y' + q(t)y = 0$$

    where p and q are continuous on $\mathbf{R}$.

12. (10 points) Use the method of successive approximations to compute the approximation $\phi_3(t)$ to the initial value problem

$$y' = 3ty - t^2, \qquad y(0) = 0$$

    starting with the initial approximation $\phi_0(t) = 0$.

13. (10 points) Give a closed form expression for the solution to the first order difference equation

$$y_{n+1} = 2y_n + n, \qquad y_0 = 1$$

14. (10 points) Find the general solution to

$$t^2 y'' - 2y = 3t^2$$

    for $t > 0$ given that the general solution to the homogeneous equation $t^2 y'' - 2y = 0$ for $t > 0$ is

$$C_1 t^2 + C_2 t^{-1}.$$